Lecture
on
Sub: Fluid Machines
Code : MEC 206
Topic : Compressible and Incompressible Flow

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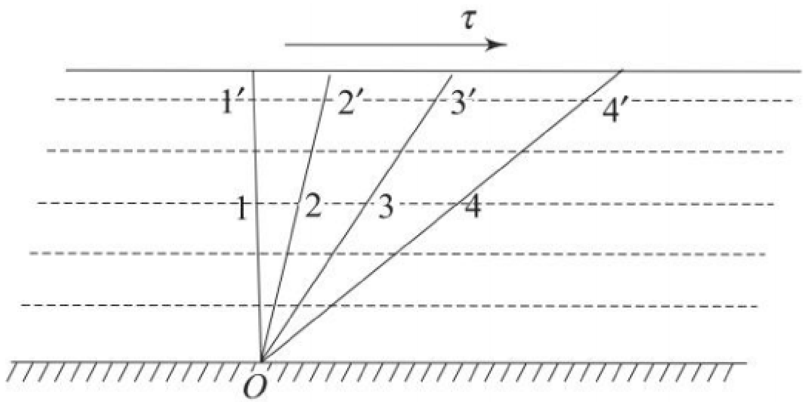
What is fluid?

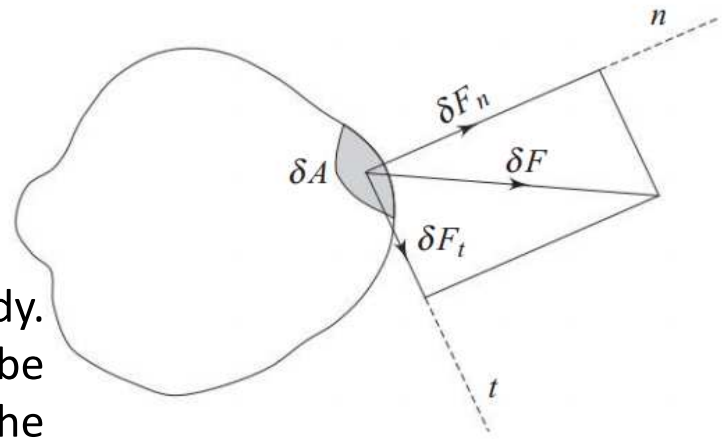
Definition of fluid

A fluid is a substance that deforms continuously when subjected to a tangential or shear stress **(τ)**, however small the shear stress may be.



The normal stress $\sigma = \lim\limits_{\delta A \Rightarrow 0} (\delta F_n / \delta A)$

and shear stress $\tau = \lim\limits_{\delta A \Rightarrow 0} (\delta F_t / \delta A)$



Let us consider a small area ***dA*** on the surface of a body. The force acting on this area is ***dF***. This force can be resolved into two components, namely, $\boldsymbol{dF_n}$ along the normal to the area ***dA*** and $\boldsymbol{dF_t}$ along the plane of ***dA***. $\boldsymbol{dF_n}$ and $\boldsymbol{dF_t}$ are called the normal and tengential forces respectively.

A solid body undergoes either a definite (say α) deformation or breaks completely when shear stress is applied on it. The amount of deformation (α) is proportional to the magnitude of applied stress up to some limiting condition. If this were an element of fluid, there would have been no fixed ‘α’ even for an infinitesimally small shear stress. It can be simply said, that while solids can resist tangential stress under static conditions, fluids can do it only under dynamic situation. Moreover, when the tangential stress disappears, solids regain either fully or partly their original shape, whereas a fluid can never regain its original shape.

What is fluid?

Concept of Continuum

The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables, i.e. assumes a continuous distribution of mass within the matter or system with no empty space.

> The most fundamental form of description of motion of a fluid is the behaviour of discrete molecules which constitute the fluid. But in liquids, molecular description is not required in order to analyse the fluid motion because the strong intermolecular cohesive forces make the entire liquid mass to behave as a continuous mass of substance.

Fluid properties

Density (ρ): The density of a fluid is its mass per unit volume. Density has the unit of kg/m^3**.**

If a fluid element enclosing a point P has a volume $\Delta \text{Ꝟ}$ and mass Δm then density (ρ) at point P is written as

$$\rho = \lim_{\Delta \text{Ꝟ} \to 0} \frac{\Delta m}{\Delta \text{Ꝟ}} = \left. \frac{dm}{d\text{Ꝟ}} \right|_P$$

Specific Weight (Ɏ) The specific weight is the weight of fluid per unit volume (ρg). In SI units, Ɏ will be expressed in N/m^3.

Specific Volume (ϑ) The specific volume of a fluid is the volume occupied by unit mass of fluid (1/ρ) in m^3/kg.

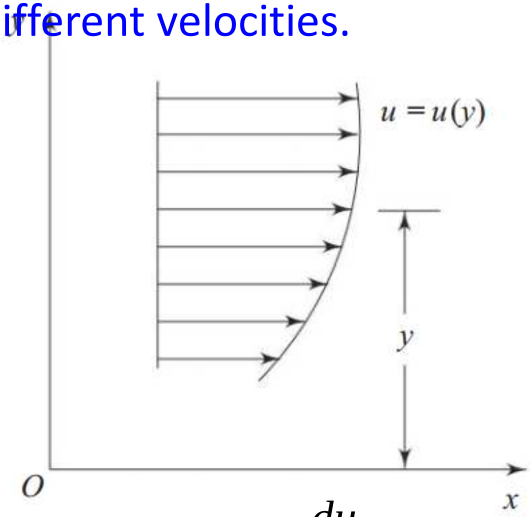
Specific Gravity (s) is the ratio of density of a liquid at actual conditions to the density of pure water at 4 °C. The specific gravity of a gas is the ratio of its density to that of either hydrogen or air at some specified temperature or pressure. However, the conditions must be stated while referring to the specific gravity of a gas.

What is fluid?

Fluid properties

Viscosity (μ) is a property of the fluid that depends on its state, especially, when the fluid is in motion. The causes of viscosity : (i) intermolecular force of cohesion (ii) molecular momentum exchange. When the fluid elements move with different velocities, each element will feel some resistance due to fluid friction within the elements. Therefore, shear stresses can be identified between the fluid elements with different velocities.

The viscosity of a fluid is a measure of its resistance to deformation at a given rate. For liquids, it corresponds to the informal concept of "thickness".



$$\tau = \mu \frac{du}{dy}$$

Let, in a fluid flow the upper layer, which is moving faster, tries to draw the lower slowly moving layer along with it by means of a force F along the direction of flow on this layer. Similarly, the lower layer tries to retard the upper one, according to Newton's third law, with an equal and opposite force F on it. Thus, the dragging effect of one layer on the other is experienced by a tangential force F on the respective layers. If F acts over an area of contact A, then the shear stress τ is defined as $\tau = F/A$.

Newton postulated that τ is proportional to Δu/Δy, where Δy is the distance of separation of the two layers and Δu is the difference in their velocities. In the limiting case of Δy->0, Δu/Δy equals to du/dy, the velocity gradient at a point in a direction perpendicular to the direction of the motion of the layer.

where, the constant of proportionality μ is known as the **viscosity coefficient** or **dynamic viscosity** or **viscosity** (in SI Ns/m^2 and kg/ms). **It is called Newton's law of viscosity.**

What is fluid?

Fluid properties

Ideal Fluid- It is hypothetical fluid having a zero viscosity (μ = 0). The resulting motion is called as ideal or inviscid flow. All the fluids in reality have viscosity (μ > 0) and hence they are termed as **real fluid** and their motion is known as viscous flow.

However, Under certain situations of very high velocity flow of viscous fluids, an accurate analysis of flow field away from a solid surface can be made from the ideal flow theory.

Non-Newtonian Fluids There are certain fluids where the linear relationship between the shear stress and the deformation rate is not valid. For these fluids the viscosity varies with rate of deformation. The mathematical model for describing the non-Newtonian fluids is the Power-Law model which is also known as Ostwald-de Waele model.

$$\tau = m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}$$

where m is known as the flow consistency index and n is the flow behaviour index.

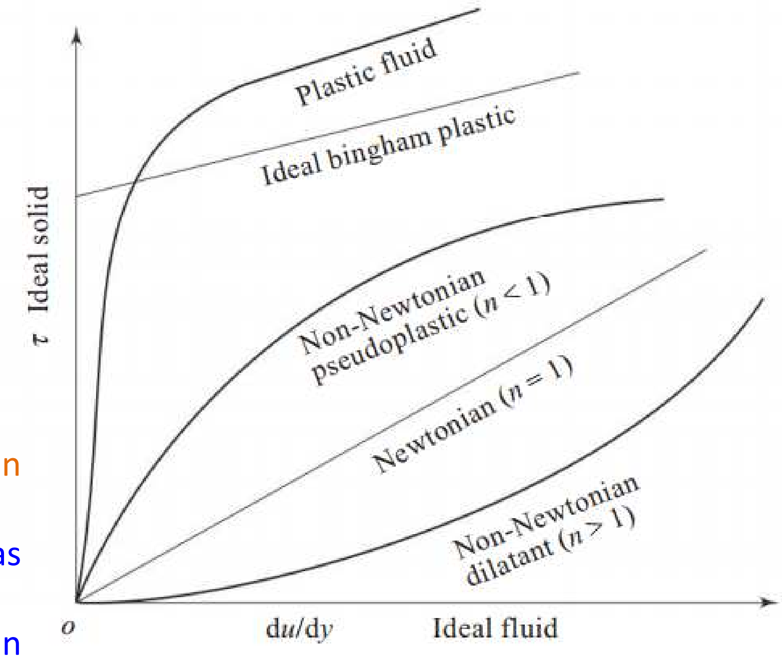
$$\mu = \left|\frac{du}{dy}\right|^{n-1}$$

Here viscosities of non-Newtonian fluids are function of deformation rate and are often termed as apparent or effective viscosity.

When n = 1, m equals to μ, the model identically satisfies Newtonian model as a special case.

When n < 1, the model is valid for pseudoplastic fluids, such as gelatine, blood, milk etc.

When n > 1, the model is valid for dilatant fluids, such as sugar in water, aqueous suspension of rice starch etc.



What is fluid?

Fluid properties

Gelatine, n <1

Corn starch, n >1

lava, n >1

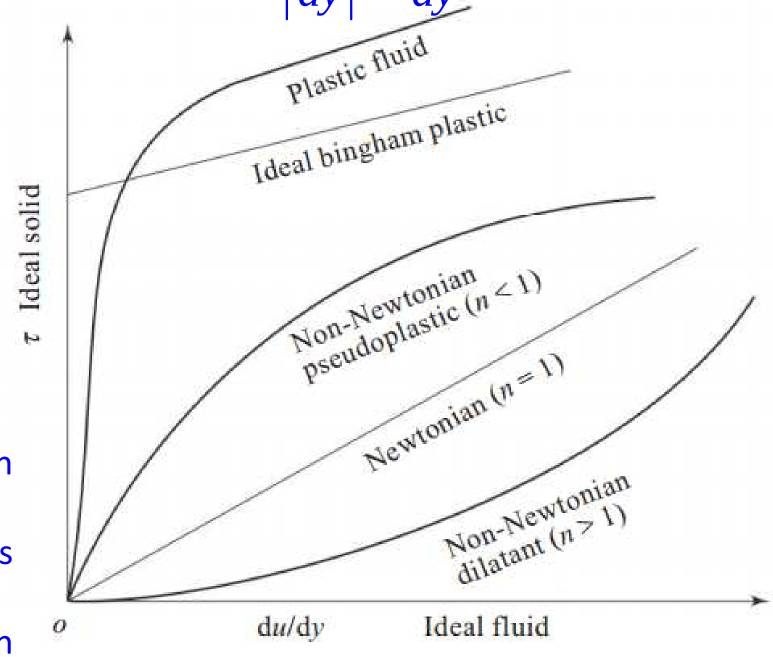
A Bingham plastic is a viscoplastic material that behaves as a rigid body at low stresses but flows as a viscous fluid at high stress. Ex: slurries

Bingham plastic

$$\tau = \tau_y + m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}, n = 1, m = \mu$$

Bingham plastic, slurry mud flow through pipe

Slurry pump



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What is fluid?

Fluid properties

Kinematic Viscosity is a measure of the internal resistance to flow of a fluid. $\nu = \mu / \rho$

Dynamic or absolute viscosity is the is a measure a fluid's resistance to flow when an external force is applied. Kinematic viscosity is under the weight of gravity.

Its dimensional formula is L^2T^{-1} and is expressed as m^2/s in SI units. The importance of kinematic viscosity in practice is realised due to the fact that while the viscous force on a fluid element is proportional to **μ**, the inertia force is proportional to **ρ** and this ratio of **μ** and **ρ** appears in several dimensionless similarity parameters like Reynolds number **VL/ν**, Prandtl number **ν/α** etc. in describing various physical problems.

Measuring Kinematic Viscosity

There are several ways to find the kinematic viscosity of a fluid, but the most common method is determining the time it takes a fluid to flow through a capillary tube. The time is converted directly to kinematic viscosity using a calibration constant provided for the specific tube. **The unit of measure of kinematic viscosity is Centistokes (cSt).**

When Should You Use Dynamic Viscosity Measurements?

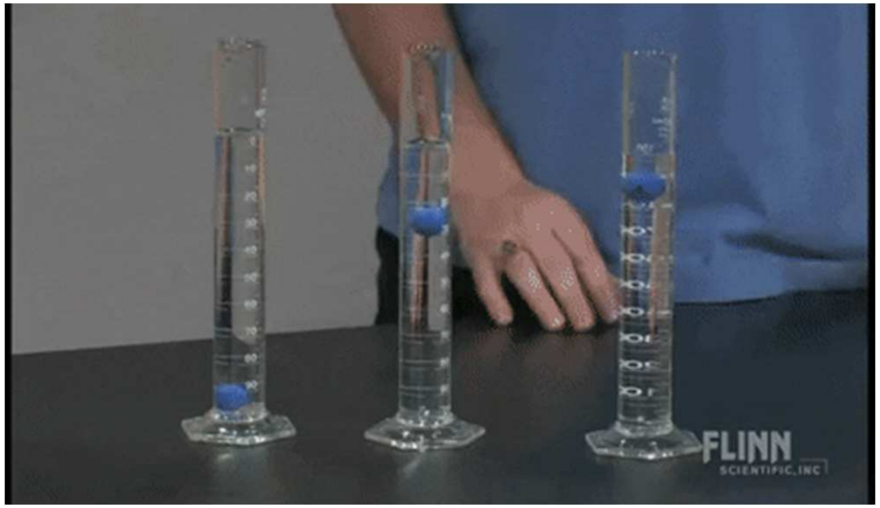
The formula for dynamic or absolute viscosity is 1 centipoise (cP) equals 1 millipascal-second (mPa-s). Pascal is a unit of force just like horsepower. Therefore, this type of viscosity measurement requires an external force in order to be measured.

No-slip Condition of Viscous Fluids When a viscous fluid flows over a solid surface, the fluid elements adjacent to the surface attain the velocity of the surface; the relative velocity between the solid surface and the adjacent fluid particles is zero, i.e. no-slip condition.

What is fluid?

Fluid properties

Students calculate the viscosity of various household fluids by measuring the amount of time it takes marble or steel balls to fall given distances through the liquid



Around 1840, a French mathematician named Jean Leonard Marie Poiseuille conducted tests involving the flow of blood through small glass tubes. Poiseuille found that different blood flowed at different speeds through the glass tubes with the same amount of force.

This led him to conclude that different fluids have an internal friction which must be overcome by an external force in order to flow. This internal friction is measured by the force needed to make it flow and was given the measurement name of poise. To make readings easier, centipoise (cP) is preferred for lubricant viscosities. The term dynamic or absolute is used for this viscosity measurement.

Direct Industry Kinematic viscosity measurement viscometer - miniAV®-LT - Cannon Instrument

About the same time Poise was performing his tests, an Irishman named Sir George Stokes was dropping particles into fluids and measuring how fast they fell to the bottom. He discovered that the same particle sank at different rates in different fluids.

Stokes surmised there was some type of internal friction in the fluid causing the different rates of falling.

1 centistoke (cSt) equals 1 millimeter squared per second (mm^2/s). This is a rate of flow. It is the time it takes to have a known amount of fluid flow a given distance.

What is fluid?

Fluid properties

Compressibility of any substance is the measure of its change in volume under the action of external forces, namely, the normal compressive forces (the force δF_n in first figure, but in the opposite direction). The normal compressive stress on any fluid element at rest is known as hydrostatic pressure p and arises as a result of innumerable molecular collisions in the entire fluid. **The degree of compressibility of a substance is characterised by the bulk modulus of elasticity (E)**. The negative sign in equation indicates that an increase in pressure is associated with a decrease in volume.

$$E = \lim_{\Delta \forall \to 0} \frac{-\Delta p}{\Delta \forall / \forall}$$

For a given mass of a substance, the change in its volume and density satisfies the relation $\frac{\Delta \forall}{\forall} = \frac{-\Delta \rho}{\rho}$

Thus, E can be expressed as $E = \lim_{\Delta \rho \to 0} \frac{\Delta p}{\Delta \rho / \rho} = \rho \frac{dp}{d\rho}$

Values of E for liquids are very high as compared with those of gases (except at very high pressures). Therefore, liquids are usually termed as incompressible fluids though, in fact, no substance is theoretically incompressible. For example, the bulk modulus of elasticity for water and air at atmospheric pressure are approximately 2×10^6 kN/m^2 and 101 kN/m^2 respectively. It indicates that air is about 20,000 times more compressible than water. Hence water can be treated as incompressible.

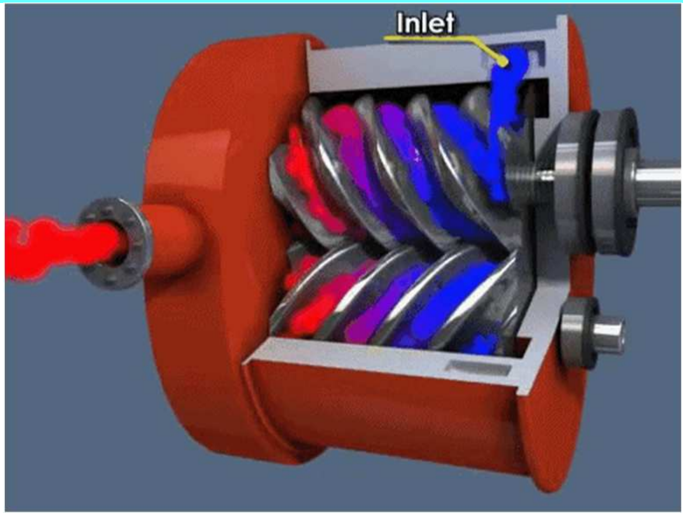
What is fluid?

Fluid properties

Compressibility K, is usually defined for gases. It is the reciprocal of **E** as

$$K = \frac{1}{\rho}\frac{d\rho}{dp} = -\frac{1}{\forall}\left(\frac{d\forall}{dp}\right)$$

K is often expressed in terms of specific volume **(ϑ)** . For any gaseous substance, a change in pressure is generally associated with a change in volume and a change in temperature simultaneously. A functional relationship between the pressure, volume and temperature at any equilibrium state is known as thermodynamic equation of state for the gas. For an ideal gas, the thermodynamic equation of state is given by **$p=\rho RT$**



Screw compressor

where T is the temperature in absolute thermodynamic or gas temperature scale (which are, in fact, identical), and R is known as the characteristic gas constant, the value of which depends upon a particular gas.

However, this equation is also valid for the real gases which are thermodynamically far from their liquid phase. For air, the value of R is 287 J/kg K. The relationship between the pressure **p** and the volume **$\forall$** for any process undergone by a gas depends upon the nature of the process. A general relationship is usually expressed in the form of **$p\forall^x$=const**.

For a constant temperature (isothermal) process of an ideal gas, **x** = 1. If there is no heat transfer to or from the gas, the process is known as adiabatic. A frictionless adiabatic process is called an isentropic process and **x** equals to the ratio of specific heat at constant pressure to that at constant volume.

What is fluid?

Fluid properties

A general relationship is usually expressed in the form of

$p\cancel{V}^{x} = const$ After differentiation $\dfrac{d\cancel{V}}{dp} = -\dfrac{\cancel{V}}{xp}$

Using equation of compressibility (K) and modulus of elasticity (E), the above relationship can be rewritten as

$E = xp$ $K = \dfrac{1}{xp}$

$$\boxed{E = \rho \frac{dp}{d\rho} \qquad K = \frac{1}{\rho}\frac{d\rho}{dp} = -\frac{1}{\cancel{v}}\left(\frac{d\cancel{v}}{dp}\right)}$$

Therefore, the compressibility K, or bulk modulus of elasticity E for gases depends on the nature of the process through which the pressure and volume change. For an isothermal process of an ideal gas (x = 1), E = p or K = 1/p. The value of E for air quoted earlier is the isothermal bulk modulus of elasticity at normal atmospheric pressure and hence the value equals to the normal atmospheric pressure.

What is fluid?

Fluid properties

Distinction between an Incompressible and a Compressible Flow

In order to know whether it is necessary to take into account the compressibility of gases in fluid flow problems, we have to consider whether the change in pressure brought about by the fluid motion causes large change in volume or density.

Bernoulli's equation: $p+\frac{1}{2}\rho V^2 = const$

$$\boxed{E=\rho\frac{dp}{d\rho}} \Rightarrow \frac{\Delta\rho}{\rho}\approx\frac{dp}{E} \Rightarrow \frac{\Delta\rho}{\rho}\approx\frac{1}{2}\frac{\rho V^2}{E}$$

$$\frac{dp}{d\rho}+\frac{1}{2}V^2=0 \Rightarrow dp=-\frac{1}{2}V^2 d\rho \Rightarrow \frac{dp}{E}=-\frac{1}{2}V^2\frac{d\rho}{E}$$

Now, we can say that if **(Δρ/ρ)** is very small, the flow of gases can be treated as incompressible with a good degree of approximation.

According to Laplace equation, the velocity of sound is given by $a=\sqrt{E/\rho}$

$$\frac{\Delta\rho}{\rho}\approx\frac{1}{2}\frac{V^2}{a^2}$$

where **Ma** is the ratio of the velocity of flow to the acoustic velocity in the flowing medium at the condition and is known as Mach number.

From the aforesaid argument, it is concluded that the compressibility of gas in a flow can be neglected if **(Δρ/ρ)** is considerably smaller than unity **(<< 1).**

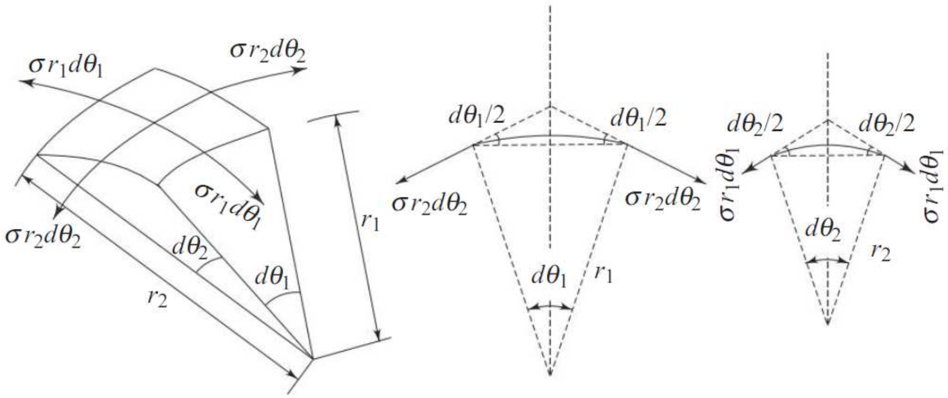
In other words, if the flow velocity is small as compared to the local acoustic velocity, compressibility of gases can be neglected. Considering a maximum relative change in density of 5 percent as the criterion of an incompressible flow, the upper limit of Mach number becomes approximately 0.33. In case of air at standard pressure and temperature, the acoustic velocity is about 335.28 m/sec. Hence a Mach number of 0.33 corresponds to a velocity of about 110 m/sec. Therefore flow of air up to a velocity of 110 m/sec under standard condition can be considered as incompressible flow.

What is fluid?

Fluid properties

Surface Tension of Liquids The phenomenon of surface tension arises due to the two kinds of intermolecular forces (i) cohesion and (ii) adhesion. The force of attraction between the molecules of a liquid by virtue of which they are bound to each other to remain as one assemblage of particles is known as the force of cohesion. This property enables the liquid to resist tensile stress. On the other hand, the force of attraction between unlike molecules, i.e. between the molecules of different liquids or between the molecules of a liquid and those of a solid body when they are in contact with each other, is known as the force of adhesion. This force enables two different liquids to adhere to each other or a liquid to adhere to a solid body or surface.

The dimensional formula is F/L or MT^{-2}. It is usually expressed in N/m in SI units. Surface tension is a binary property of the liquid and gas or two liquids which are in contact with each other and define the interface. It decreases slightly with increasing temperature. The surface tension of water in contact with air at 20 °C is about 0.073 N/m.



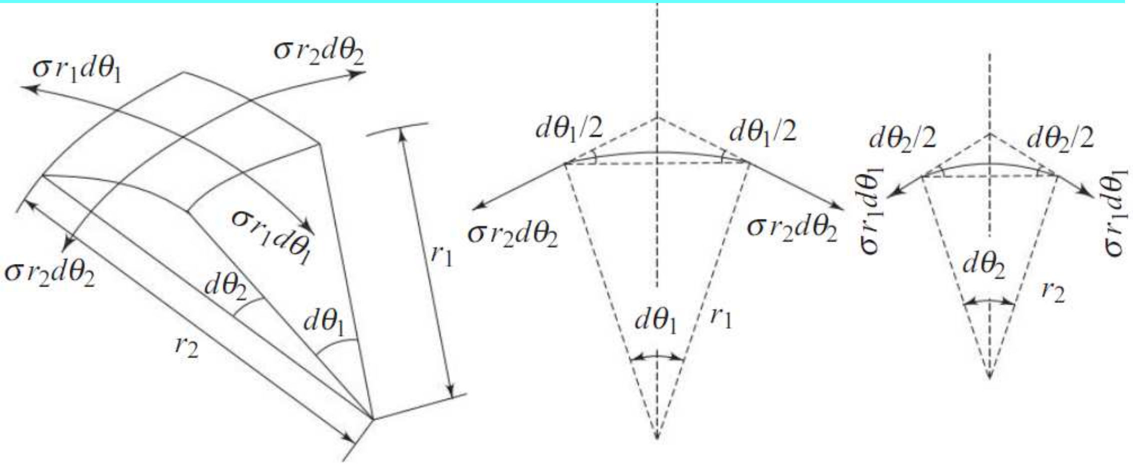
Considering the equilibrium of this small elemental surface, a force balance in the direction perpendicular to the surface results.

What is fluid?

Fluid properties

Surface Tension of Liquids

Consider an elemental curved liquid surface separating the bulk of liquid in its concave side and a gaseous substance or another immiscible liquid on the convex side.



$$2\sigma r_2 d\theta_2 \sin\left(\frac{d\theta_1}{2}\right) + 2\sigma r_1 d\theta_1 \sin\left(\frac{d\theta_2}{2}\right) = \left(p_i - p_0\right) r_1 r_2 d\theta_1 d\theta_2$$

For small angles $\sin\left(\frac{d\theta_1}{2}\right) = \frac{d\theta_1}{2}$ and $\sin\left(\frac{d\theta_2}{2}\right) = \frac{d\theta_2}{2}$

Hence, from the above equation of force balance we can write (dividing by $r_1 r_2 d\vartheta_1 d\vartheta_2$)

$$\frac{\sigma}{r_1} + \frac{\sigma}{r_2} = \left(p_i - p_0\right) = \Delta p$$

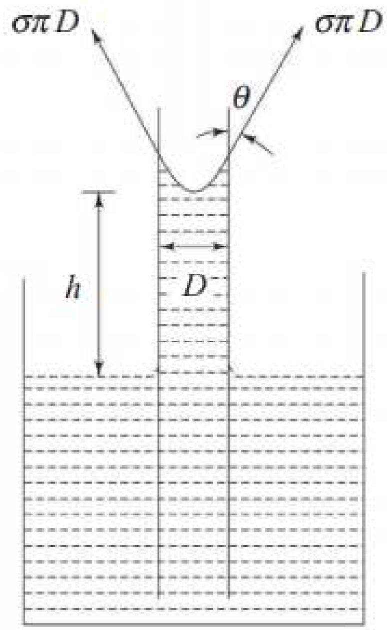
and σ is the surface tension of the liquid in contact with the specified fluid at its convex side. If the liquid surface coexists with another immiscible fluid, usually gas, on both the sides, the surface tension force appears on both the concave and convex interfaces and the net surface tension force on the surface will be twice as that described.

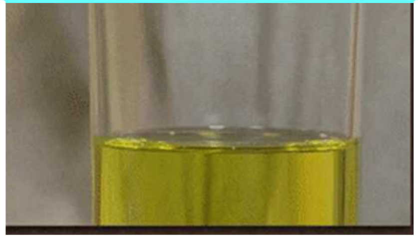
What is fluid?

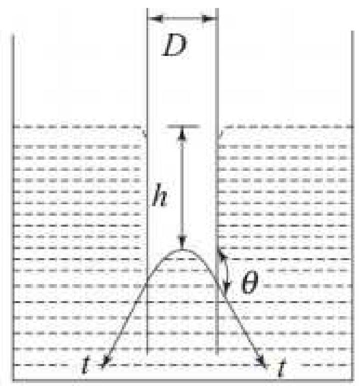
Fluid properties

Capillarity

When a liquid is in contact with a solid, if the forces of adhesion between the molecules of the liquid and the solid are greater than the forces of cohesion among the liquid molecules themselves, the liquid molecules crowd towards the solid surface. The area of contact between the liquid and solid increases and the liquid thus wets the solid surface. The reverse phenomenon takes place when the force of cohesion is greater than the force of adhesion. These adhesion and cohesion properties result in the phenomenon of capillarity by which a liquid either rises or falls in a tube dipped into the liquid depending upon whether the force of adhesion is more than that of cohesion or not. The angle θ, is the area wetting contact angle made by the interface with the solid surface.







Capillary rise

Adhesion > cohesion
liquid wets the surface

Capillary depression

Adhesion < cohesion
liquid stays away from the surface

Equating the weight of the column of liquid h with the vertical component of the surface tension force

$$\frac{\pi D^2}{4} h\rho g = \sigma\pi D\cos\theta \Rightarrow h = \frac{4\sigma\cos\theta}{\rho g D}$$

For pure water in contact with air in a clean glass tube, the capillary rise takes place with q = 0. Mercury causes capillary depression with an angle of contact of about 130° in a clean glass in contact with air. Since h varies inversely with D, an appreciable capillary rise or depression is observed in tubes of small diameter only.

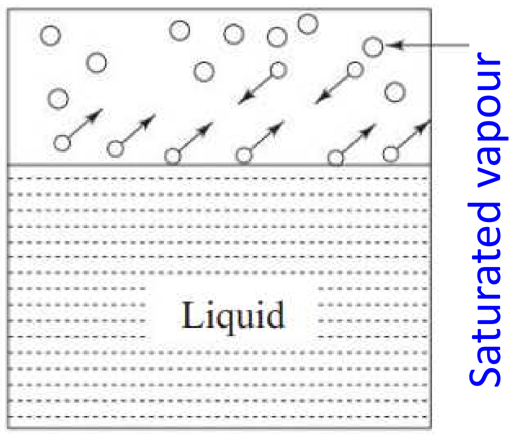
What is fluid?

Fluid properties

Vapour Pressure

All liquids have a tendency to evaporate when exposed to a gaseous atmosphere. The rate of evaporation depends upon the molecular energy of the liquid which in turn depends upon the type of liquid and its temperature. The vapour molecules exert a partial pressure in the space above the liquid, known as vapour pressure. If the space above the liquid is confined and the liquid is maintained at constant temperature, after sufficient time, the confined space above the liquid will contain vapour molecules to the extent that some of them will be forced to enter the liquid. Eventually an equilibrium condition will evolve when the rate at which the number of vapour molecules striking back the liquid surface and condensing is just equal to the rate at which they leave from the surface. The space above the liquid then becomes saturated with vapour.

The vapour pressure of a given liquid is a function of temperature only and is equal to the saturation pressure for boiling corresponding to that temperature. Hence, the vapour pressure increases with the increase in temperature. Therefore the phenomenon of boiling of a liquid is closely related to the vapour pressure. In fact, when the vapour pressure of a liquid becomes equal to the total pressure impressed on its surface, the liquid starts boiling.



This concludes that boiling can be achieved either by raising the temperature of the liquid, so that its vapour pressure is elevated to the ambient pressure, or by lowering the pressure of the ambience (surrounding gas) to the liquid vapour pressure at the existing temperature.

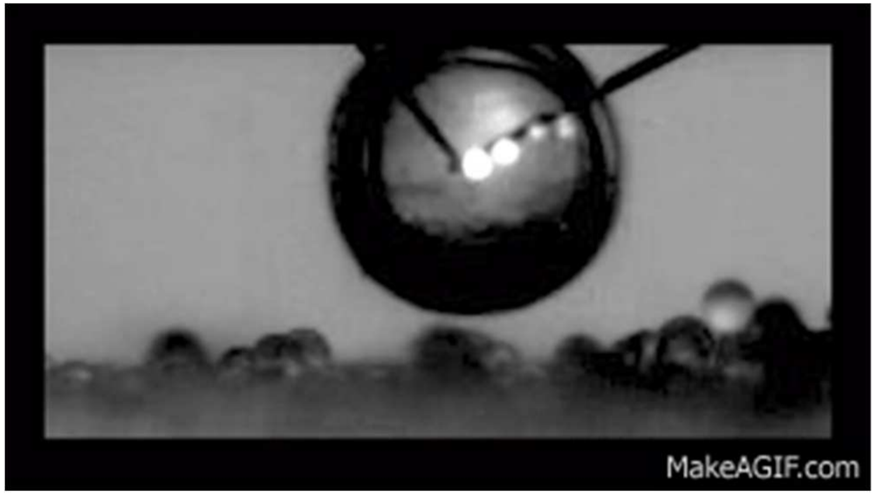
What is fluid?

Fluid properties

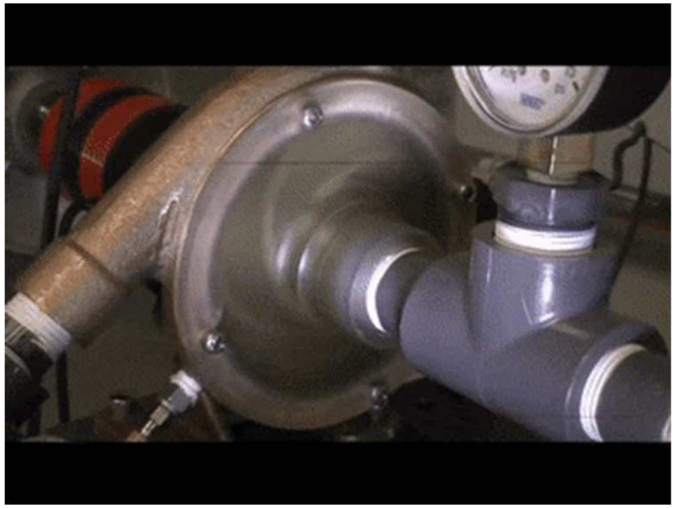
Vapour Pressure → Cavitation in fluid machines

propeller cavitation

Axiom DWS type propeller cavitation testing



Cavitation bubbles bursting with cleaning power



Cavitation in pump

✓ **Kinematics of Fluid**

Kinematics of Fluid

Kinematics is the geometry of motion. Therefore the kinematics of fluid is that branch of fluid mechanics which describes the fluid motion and its consequences without consideration of the nature of forces causing the motion.

FLOW FIELD AND DESCRIPTION OF FLUID MOTION

A flow field is a region in which the flow is defined at each and every point at any instant of time. Usually, velocity describes the flow. In other words, a flow field is specified by the velocities at different points in the region at different times.

Fluid motion is described by two methods discussed as follows:

Lagrangian Method In this method, the fluid motion is described by tracing the kinematic behaviour of each and every individual particle constituting the flow.

Identities of the particles are made by specifying their initial position (spatial location) at a given time. The position of a particle at any other instant of time then becomes a function of its identity and time.

This statement can be analytically expressed as

$\vec{S} = S\left(\vec{S}_0, t\right)$ This is the expression of position vector of a particle (with respect to a fixed point of reference) at a time t.

Kinematics of Fluid

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Eulerian Method (by Leonhard Euler) avoids the determination of the movement of each individual fluid particle in all details. Instead it seeks the velocity $\vec{V}$ and its variation with time t at each and every location $\vec{S}$ in the flow field.

While in the Lagrangian view, all hydrodynamic parameters are tied to the particles or elements, in Eulerian view, they are functions of location and time.

Mathematically, the flow field in Eulerian method is described as

$$\vec{V} = V\left(\vec{S}, t\right)$$

$$\vec{V} = \vec{i}u + \vec{j}v + \vec{k}w$$

$$\vec{S} = \vec{i}x + \vec{j}y + \vec{k}z$$

Kinematics of Fluid

Steady and Unsteady Flows A steady flow is defined as a flow in which the various hydrodynamic parameters and fluid properties at any point do not change with time. Flow in which any of these parameters changes with time is termed as unsteady flow. In Eulerian approach, a steady flow is described as, $\vec{V} = V\left(\vec{S}, \cancel{t}\right) = V\left(\vec{S}\right)$

Uniform and Non-uniform Flow When velocity and other hydrodynamic parameters (like pressure and density), at any instant of time do not change from point to point in a flow field, the flow is said to be uniform. If, however, changes do occur from one point to another, the flow is non-uniform. Hence, for a uniform flow, the velocity is a function of time only, which can be expressed in Eulerian description as $\vec{V} = V\left(t\right)$

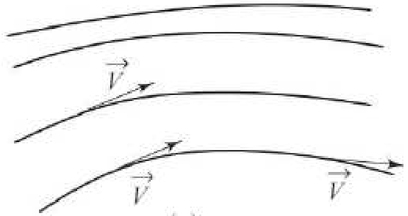
Type	*Example*
1. Steady uniform flow	Flow at constant rate through a duct of uniform cross-section. (The region close to the walls of the duct is however disregarded.)
2. Steady non-uniform flow	Flow at constant rate through a duct of non-uniform cross-section (tapering pipe.)
3. Unsteady uniform flow	Flow at varying rates through a long straight pipe of uniform cross-section. (Again the region close to the walls is ignored.)
4. Unsteady non-uniform flow	Flow at varying rates through a duct of non-uniform cross-section. Real river flow

Kinematics of Fluid

Streamlines It is the analytical description of flow velocities by the Eulerian approach. If for a fixed instant of time, a space curve is drawn so that it is tangent everywhere to the velocity vector, then this curve is called a streamline.

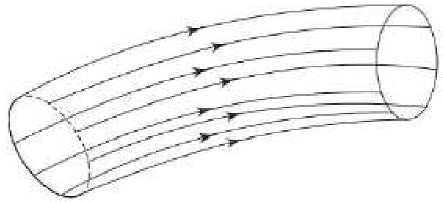
In other words, a streamline at any instant can be defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the direction of the instantaneous velocity at that point.

In a steady flow, the orientation or the pattern of streamlines will be fixed. So streamline can be written as $\vec{V} \times d\vec{S} = 0$



(a)

Fig. 3.2a Streamlines



(b)

Fig. 3.2b Stream tube

Path Lines Path lines are the outcome of the Lagrangian method in describing fluid flow and show the paths of different fluid particles as a function of time. In other words, a path line is the trajectory of a fluid particle of fixed identity.

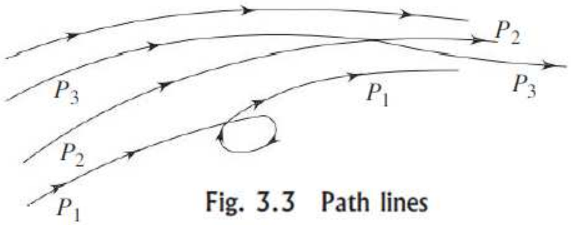
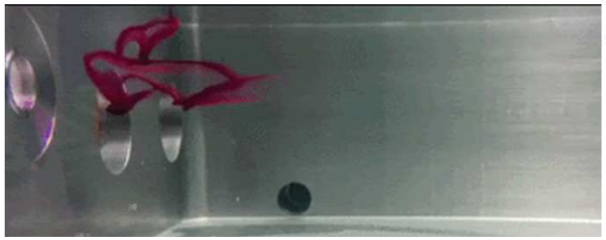
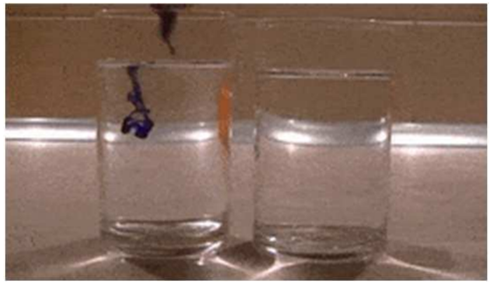


Fig. 3.3 Path lines

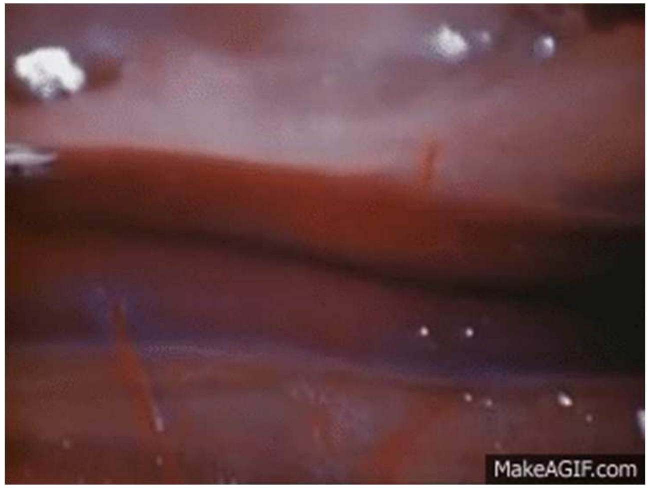
If dye is injected into a liquid at a fixed point in the flow field, then at a later time t, the dye will indicate the end points of the pathlines of particles which have passed through the injection point.



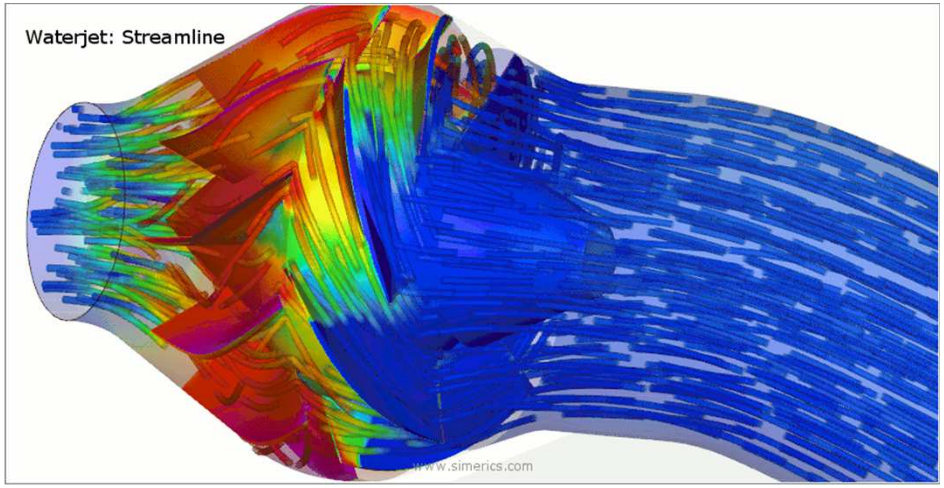
Streak Lines A streak line at any instant of time is the locus of the temporary locations of all particles that have passed though a fixed point in the flow field. While a path line refers to the identity of a fluid particle, a streak line is specified by a fixed point in the flow field.



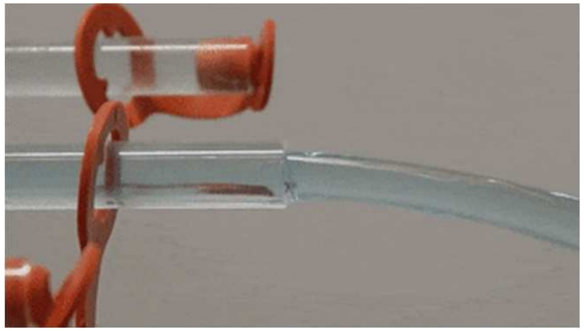
Kinematics of Fluid



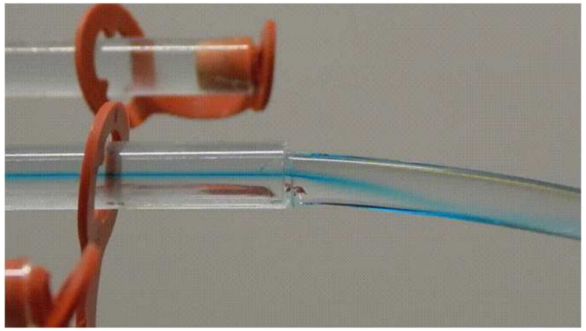
Streamline Flow in Veins



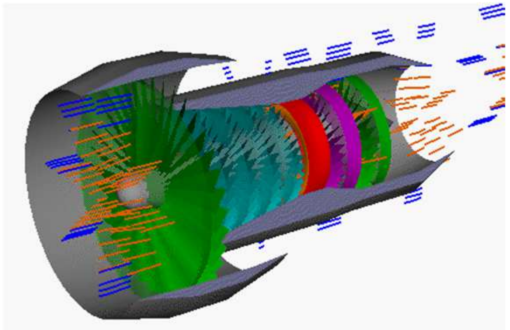
Streamline in a propeller



Laminar flow



From Turbulent to Transitional to Laminar



Streamline in a Turbofan

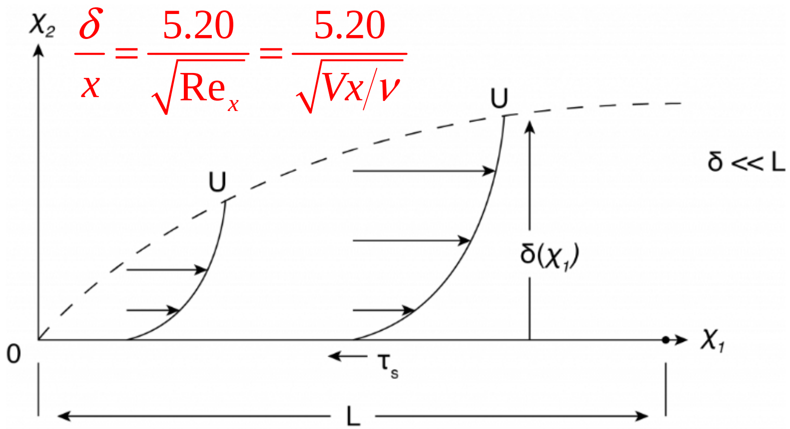
Boundary layer

Boundary layer, in fluid mechanics, thin layer of a flowing gas or liquid in contact with a surface such as that of an airplane wing or of the inside of a pipe. The flow in such boundary layers is generally laminar at the leading or upstream portion and turbulent in the trailing or downstream portion.

A boundary layer flow is the region of a larger flow field that is next to the surface and has significant effects of wall frictional forces. Since the region of interest is near the surface and the surface is assumed to be impervious / unaffected to the flow, then the velocity is nearly parallel to the surface.

Flow is from left to right and there is an upstream "leading edge" where the surface begins. At the leading edge (defined as the coordinate system origin), the flow immediately next to the surface begins to experience frictional forces due to the no slip boundary condition.

The fluid outside of this thin layer is directly unaffected by the wall friction. As the flow goes downstream the slower moving fluid near the surface exerts frictional forces on the fluid further away from the surface. This process is one of diffusion of momentum loss in a direction normal to the surface that results in a reduction of the local fluid velocity. The rate of diffusion depends on the viscosity of the fluid.



$$\frac{\delta}{x} = \frac{5.20}{\sqrt{\mathrm{Re}_x}} = \frac{5.20}{\sqrt{Vx/\nu}}$$

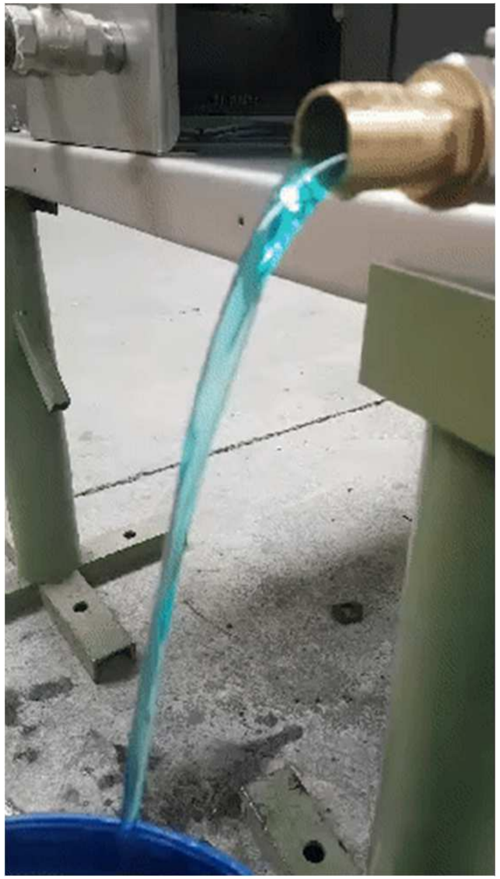
As this happens the region slowed by friction grows such that the thickness of the boundary layer, δ, increases along the flow direction. At the outer edge of this viscous region, where the flow is no longer affected or slowed by the surface generated friction, the velocity is called the "freestream" value.

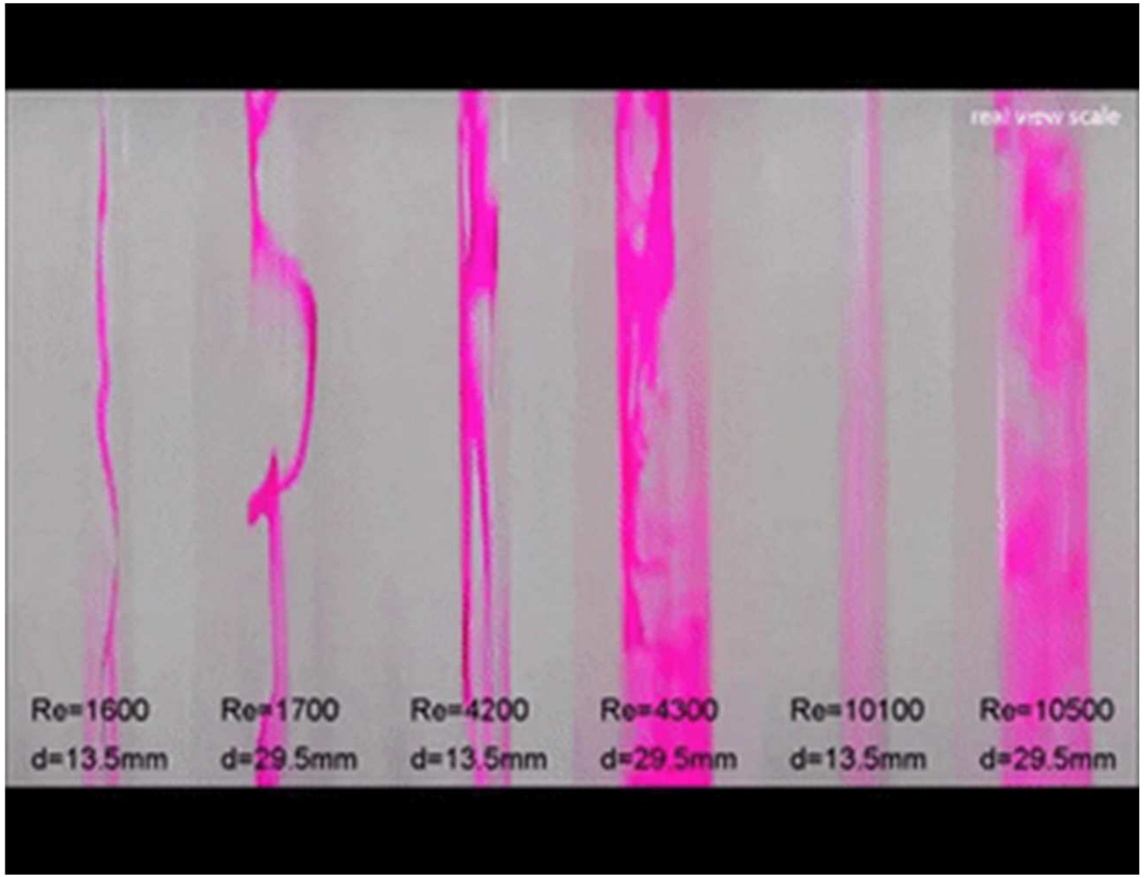
Laminar and Turbulent flow

- **Laminar flow** or streamline **flow** in pipes (or tubes) occurs when a fluid **flows** in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing,
- **Turbulent flow** is a **flow** regime characterized by chaotic property changes. Flows at Reynolds numbers larger than 4000 are typically (but not necessarily) turbulent, while those at low Reynolds numbers below 2300 usually remain laminar. Flow in the range of Reynolds numbers 2300 to 4000 and known as transition.

Reynolds number is the ratio of **inertial forces to viscous forces.**

If viscous force increases, Re decreases, i.e. close to laminar flow.

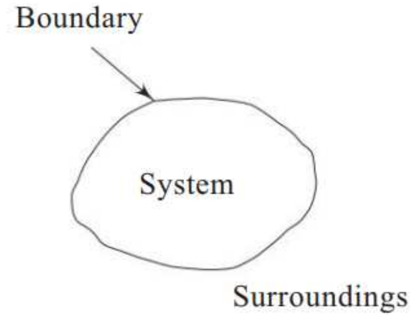




- ✓ **System**
- ✓ **Conservation Equations**

Conservation Equations and Analysis of Finite Control Volumes

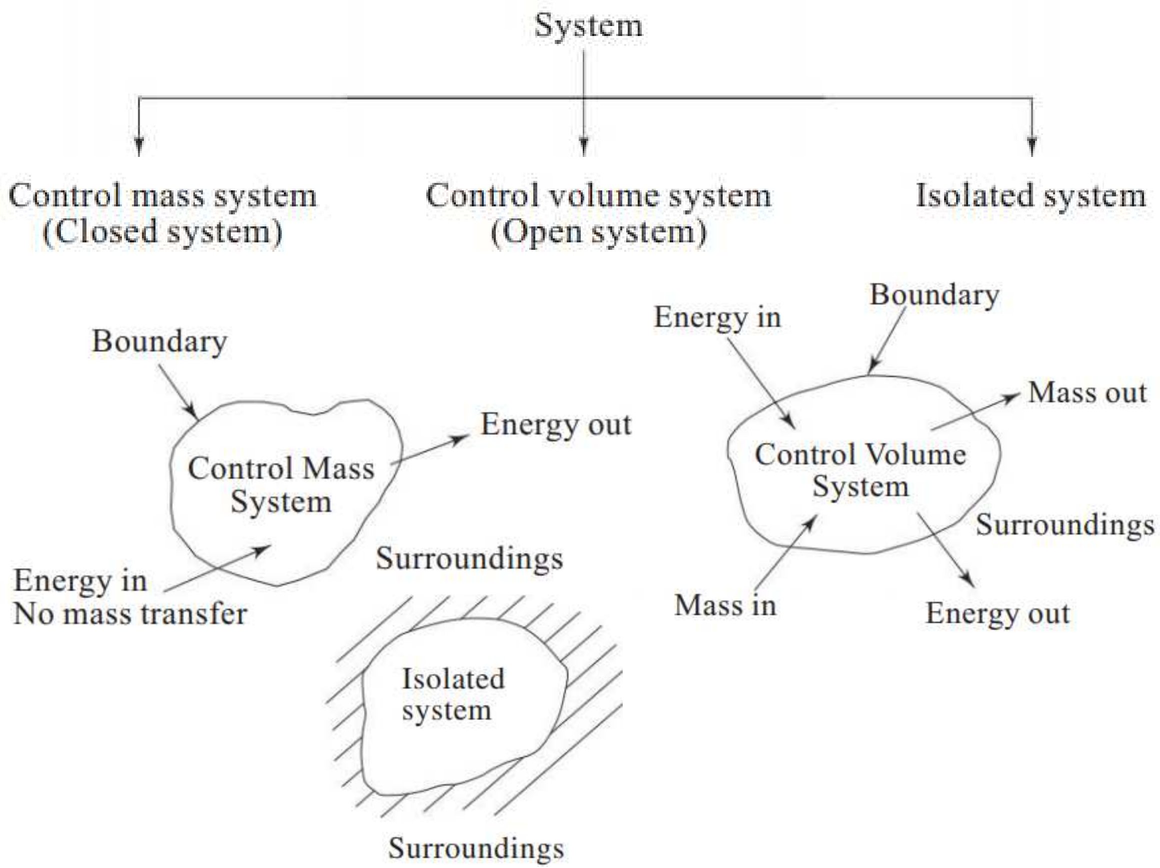
- System
- Conservation of Mass The Continuity Equation
- Conservation of Momentum: Momentum Theorem
- Analysis of Finite Control Volumes
- Euler’s Equation: The Equation of Motion for an Ideal Flow
- Conservation of Energy



A system is defined as a quantity of matter in space upon which attention is paid in the analysis of a problem. Everything external to the system is called the surroundings.

A control mass or closed system is characterised by a fixed quantity of mass of a given identity, while in an open system or control volume mass may change continuously due to the flow of mass across the system boundary.

An isolated system is one in which there is neither interaction of mass nor energy between the system and the surroundings. Therefore it is of fixed mass with same identity and fixed energy.



Conservation Equations and Analysis of Finite Control Volumes

Conservation of mass the continuity Equation

The law of conservation of mass states that mass can neither be created nor be destroyed. Conservation of mass is inherent to the definition of a closed system (control mass).

$\Delta m / \Delta t = 0$ where m is the mass of the system.

For a control volume, the principle of conservation of mass can be stated as

Rate at which mass enters the region = Rate at which mass leaves the region + Rate of accumulation of mass in the region

Or

Rate of accumulation of mass in the control volume + Net rate of mass efflux from the control volume = 0

Continuity equation is the equation of conservation of mass in a fluid flow. The general form of the continuity equation for an unsteady compressible flow is given by the formula, where where, $\vec{V}$ is the velocity vector.

Continuity equation of compressible flow :

$$\frac{\partial \rho}{\partial t} + \nabla . \left(\rho \vec{V} \right) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

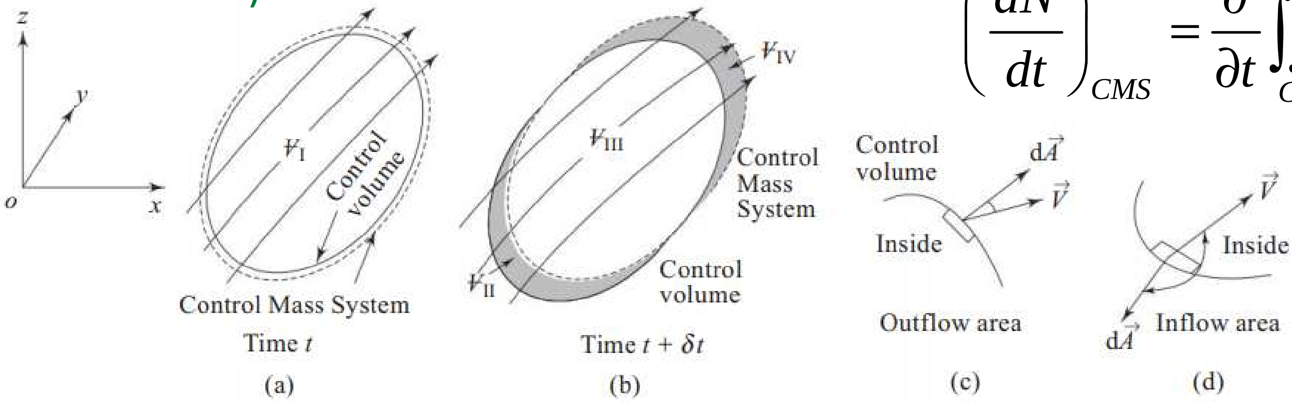
The concept of stream function is a consequence of continuity. In a two dimensional incompressible flow, the difference in stream functions between two points gives the volume flow rate (per unit width in a direction perpendicular to the plane of flow) across any line joining the points. The value of stream function is constant along a streamline.

Conservation of momentum: Momentum theorem

In Newtonian mechanics, the conservation of momentum is defined by Newton's second law of motion as follows: The rate of change of momentum of a 'body' is proportional to the impressed action and takes place in the direction of the impressed action. The momentum implied may be linear or angular and the corresponding actions are force and moment respectively. In case of fluid flow, the word body in the above statement may be substituted by the word 'particle' or 'control mass system'.

Reynolds transport theorem states the relation between equations applied to a system and those applied to a control volume. The statement of the law of conservation of momentum as applied to a control volume is known as momentum theorem. Reynolds transport theorem states that the resultant force (or torque) acting on a control volume is equal to the time rate of increase in linear momentum (or angular momentum) within the control volume plus the rate of net efflux of linear momentum (or angular momentum) from the control volume.

$$\left(\frac{dN}{dt}\right)_{CMS} = \frac{\partial}{\partial t}\iiint_{CV} \eta\rho d\forall + \iint_{CS} \eta\rho\vec{V}.d\vec{A}$$



where property N is net flow outflow from the control volume

Conservation of Mass: N=m, η=1

Conservation of Momentum of momentum : N=m$\vec{V}$, η= $\vec{V}$

Mass conservation

$$\dot{m} = \rho AV = \rho Q$$

Force: **Jet striking plate**

$$F = \rho QV$$

Conservation of momentum: Momentum theorem

Mass conservation

$$\dot{m} = \rho AV = \rho Q$$

Force: **Jet striking plate**

$$F = \rho QV$$

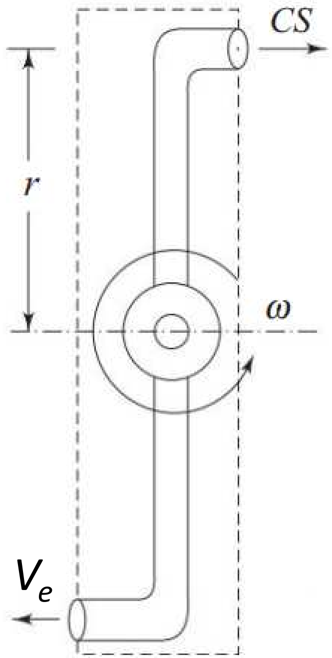
Application of momentum theorem:

(i) Forces acting due to internal flows through expanding or reducing pipe bends.

(ii) Forces on stationary and moving vanes due to impingement of fluid jets.

(iii) Jet propulsion of ship and aircraft moving with uniform velocity.

Application of moment of momentum theorem:



It is a sprinkler like turbine. The turbine rotates in a horizontal plane with angular velocity ω. The radius of the turbine is r. Water enters the turbine from a vertical pipe that is coaxial with the axis of rotation and exits through the nozzles of cross sectional area with a velocity V_e relative to the nozzle.

Application of **momentum of momentum theorem** gives

$$M_{zc} = \dot{m}\left(\vec{r} \times \vec{V}\right)$$

M_{zc} is the moment (Torque) applied to the CV.

T=Torque

Torque is a measure of the **force** that can cause an object to rotate about an axis.

$$T = F \times r$$

$$P = M_z \omega = T\omega$$

Conservation of momentum: Momentum theorem

The equation of motion (conservation of momentum) of an inviscid flow is known as **Euler's equation**.

The general form of Euler's equation is given by $\rho D\vec{V}/Dt = \rho\vec{X} - \nabla p$,
where $\vec{X}$ is the body force vector per unit mass and $\vec{V}$ is the velocity vector. Euler's equation along a streamline, with gravity as the only body force, can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X_x - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = X_y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = X_z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

In general,

$$\rho\frac{DV}{Dt} = -\frac{\partial p}{\partial s} - \rho g\frac{dz}{ds}$$

where s represents the coordinate along the streamline.

Finally, along a streamline $$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{dz}{ds}$$

Euler's equation (the equation of motion of an inviscid fluid) along a streamline for a steady flow with gravity as the only body force can be written as

$$V\frac{\partial V}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{dz}{ds}$$

Application of a force through a distance ds along the streamline would physically imply work interaction. Therefore an equation for conservation of energy along a streamline can be obtained by integrating the Eq. with respect to ds as

$$\int V\frac{\partial V}{\partial s}ds = -\int\frac{1}{\rho}\frac{\partial p}{\partial s}ds - \int g\frac{dz}{ds}ds$$

$$\frac{V^2}{2} + \int\frac{1}{\rho}dp + \int gdz = const$$ **Bernoulli's equation**

Conservation of momentum: Momentum theorem

A fluid element in motion possesses intermolecular energy, kinetic energy and potential energy. The work required by a fluid element to move against pressure is known as flow work. It is loosely termed as pressure energy. The shaft work is the work interaction between the control volume and the surrounding that takes place by the action of shear force such as the torque exerted on a rotating shaft.

The equation for conservation of energy of a steady, inviscid and incompressible flow in a conservative body force field is known as **Bernoulli's equation**. Bernoulli's equation in the case of gravity as the only body force field is given by

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=C$$

The value of C remains constant along a streamline.

The loss of mechanical energy due to friction in a real fluid is considered in Bernoulli's equation as

$$\frac{p_1}{\rho g}+\frac{V_1^{\,2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^{\,2}}{2g}+z_2+hf$$

where, *hf* is the frictional work done or loss of mechanical energy due to friction per unit weight of a fluid element while moving from station 1 to 2 along a streamline. The term *hf* is usually referred to as head loss.

A conservative force is a force with the property that the total work done in moving a particle between two points is independent of the path taken. A conservative force depends only on the position of the object. Gravitational force is an example of a conservative force, while frictional force is an example of a non-conservative force.

- ✓ **Flow of Ideal Fluids**

Flow of Ideal Fluids

Flows at high Reynolds number reveal that the viscous effects are confined within the boundary layers.

Far away from the solid surface, the flow is nearly inviscid (viscosity of the fluid is equal to zero) and in many cases it is incompressible.

Incompressible flow is a constant density flow, and we assume ρ to be constant. We visualize a fluid element of defined mass moving along a streamline in an incompressible flow.

Because the density is constant, we can write

Divergence of V

$$\nabla . V = 0 = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\left(u\hat{i} + v\hat{j} + w\hat{k}\right) = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

Over and above, if the fluid element does not rotate as it moves along the streamline, or to be precise, if its motion is translational (and deformation with no rotation) only, the flow is termed as irrotational flow.

The motion of a fluid element can in general have translation, deformation and rotation. The rate of rotation of the fluid element can be measured by the average rate of rotation of two perpendicular line segments.

The average rate of rotation about a particular axis is expressed in terms of the gradients of velocity components as

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

rotation about z-axis

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

rotation about x-axis

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

rotation about y-axis

Curl or rotation of V

$$\vec{\omega} = \frac{1}{2}\left(\nabla \times \vec{V}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(u\hat{i} + v\hat{j} + w\hat{k}\right)$$

Flow of Ideal Fluids

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Incompressible flow is a constant density flow, and we assume ρ to be constant. We visualize a fluid element of defined mass moving along a streamline in an incompressible flow.

Because the density is constant, we can write $\nabla . V = 0 = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$ Divergence of V

Imagine a pathline of a fluid particle shown in Fig. Rate of spin of the particle is w_z.

The flow in which this spin is zero throughout is known as irrotational flow. $\nabla \times \vec{V} = 0$

which demands $\vec{V} = \nabla \phi = \left(\frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k} \right)$ gradient of ϕ

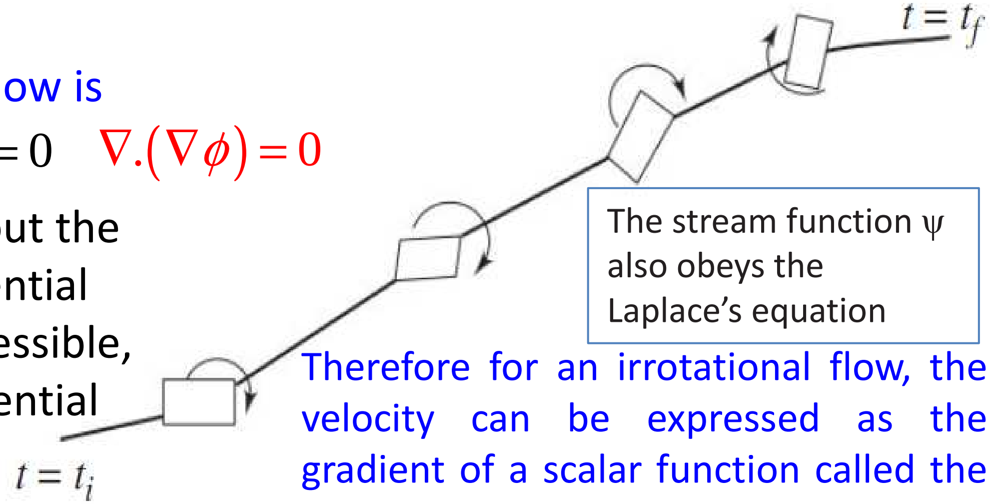
Here ϕ is potential function

Laplace's operator ∇^2

Potential flow

Inviscid, incompressible, irrotational flow is governed by Laplace's equation. $\nabla^2 \phi = 0$ $\quad \nabla . (\nabla \phi) = 0$

The analysis of Laplace's and finding out the potential functions are known as potential flow theory and the inviscid, incompressible, irrotational flow is often called as potential flow.

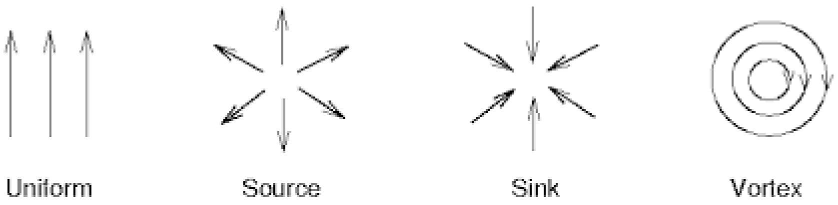


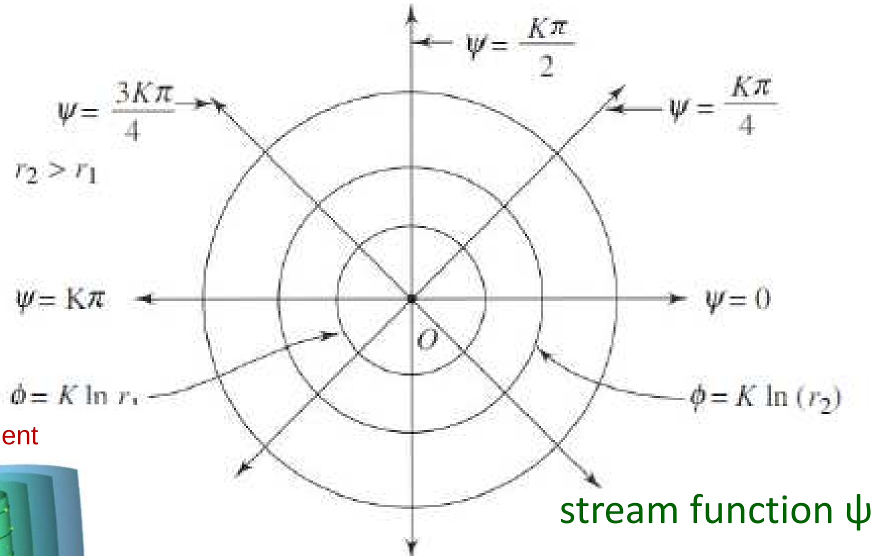
The stream function ψ also obeys the Laplace's equation

Therefore for an irrotational flow, the velocity can be expressed as the gradient of a scalar function called the velocity potential, denoted by ϕ.

Flow of Ideal Fluids

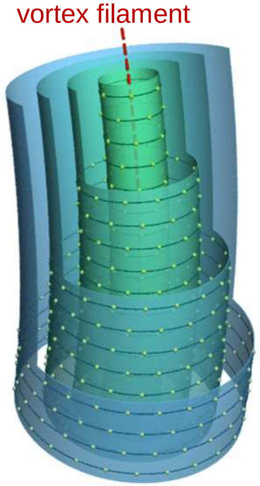
Source or Sink Consider a flow with straight streamlines emerging from a point, where the velocity along each streamline varies inversely with distance from the point. Such a flow is called source flow.





Flownet for Source/ sink

Vortex Flow In this flow all the streamlines are concentric circles about a given point where the velocity along each streamline is inversely proportional to the distance from the centre. Such a flow is called vortex (free vortex) flow. This flow is necessarily irrotational.



In a purely circulatory (free vortex flow) motion, we can write the tangential velocity as

$$v_\theta = \frac{\text{Circulation constant}}{\text{r}}$$

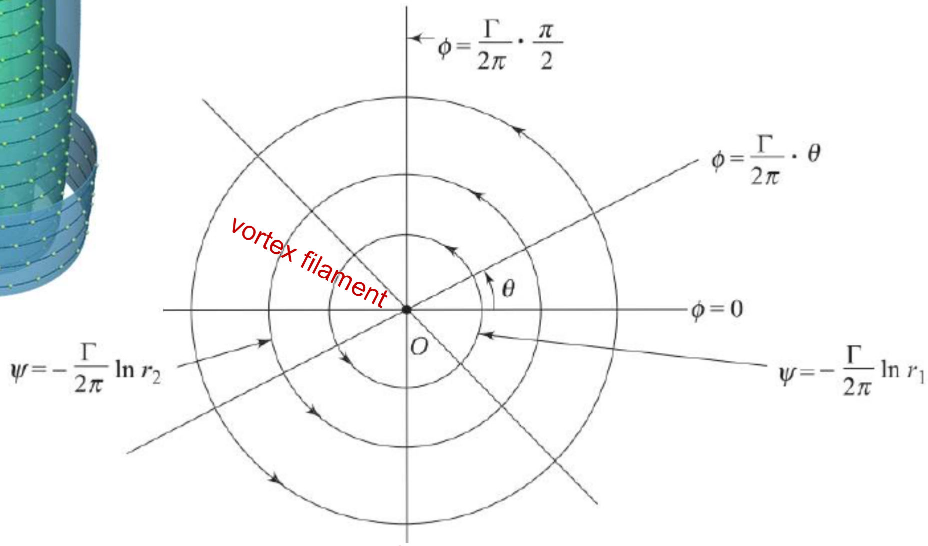
A **vortex filament** is an imaginary spatial curve that induces a rotary flow in the space through which it passes

$$v_\theta = \frac{\Gamma/2\pi}{\text{r}}$$

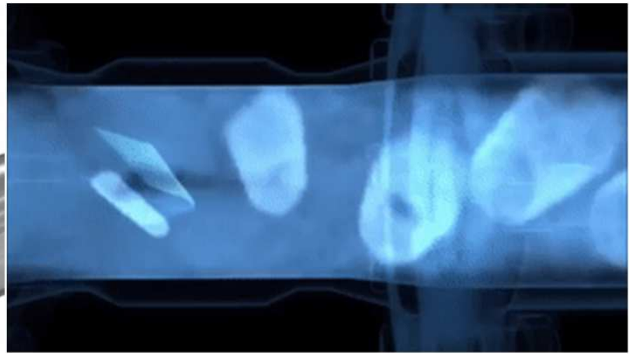
Circulation about the vortex filament

$$\Gamma = \oint \vec{V}.d\vec{s}$$

closed contour / line integral



Flownet for vortex



Vortex Flow meter

Vortex in water basin

1. Water in a tank, rotating about an axis, velocity vary directly with the radius. Line/stick is tangential or normal to the streamline will rotate just as they would in the vortex filaments
2. If velocity varies inversely with the radius, as it tend to do in the field of flow surrounding an vortex filament whether above a drain or tornedo, a line or stick in the tangential direction will tend to rotate in the opposite sense from one of the normal but at the same rate
3. Two perpendicular stick fastened together then will not rotate at all, such flow is irrotational. The central film with itself of Course is highly rotational
4. Flow in concentric circle is not the only time that is sometimes highly rotational and sometimes almost irrotational.
5. Uniform flow between parallel walls is usually rotational, on the contrary either non-uniform and unsteady flow that is rapidly accelerated is very nearly irrotational.

Flow of Ideal Fluids

The **doublet** consists of a sou
of momentum located in close
one another.

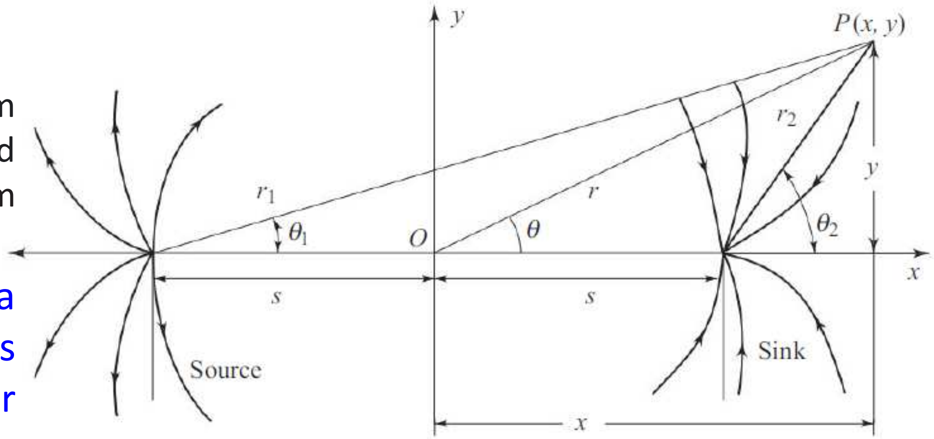
SUPERPOSITION OF ELEMENTARY FLOWS

Different flow patterns can be formed by superimposing the velocity potential and stream functions of the elementary flows stated above.

Doublet

The **doublet** consists of a source and sink of momentum located in close proximity to one another. Source and sink of equal strength K exists at equal distance s from the origin along x-axis.

Since the circulation about a singular point of a source or a sink is zero for any strength, it is obvious that the circulation about the singular point in a doublet flow must be zero.

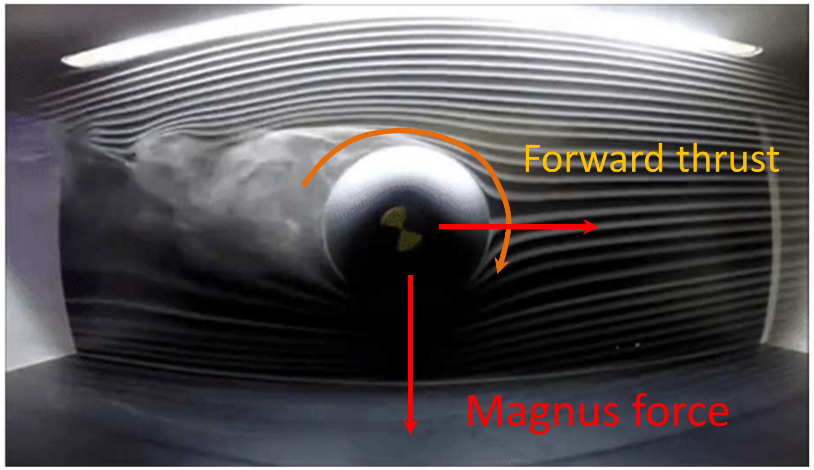


Flow About a Cylinder Without Circulation

Inviscid-incompressible flow about a cylinder in uniform flow is equivalent to the superposition of a uniform flow and a doublet.

Flow About a Rotating Cylinder

In addition to superimposed uniform flow and a doublet, a vortex is thrown at the doublet centre. We shall see that the pressure distribution will result in a force, a component of which is lift force. The phenomenon of generation of lift by a rotating object placed in a stream is known as Magnus effect.



Flow of Ideal Fluids

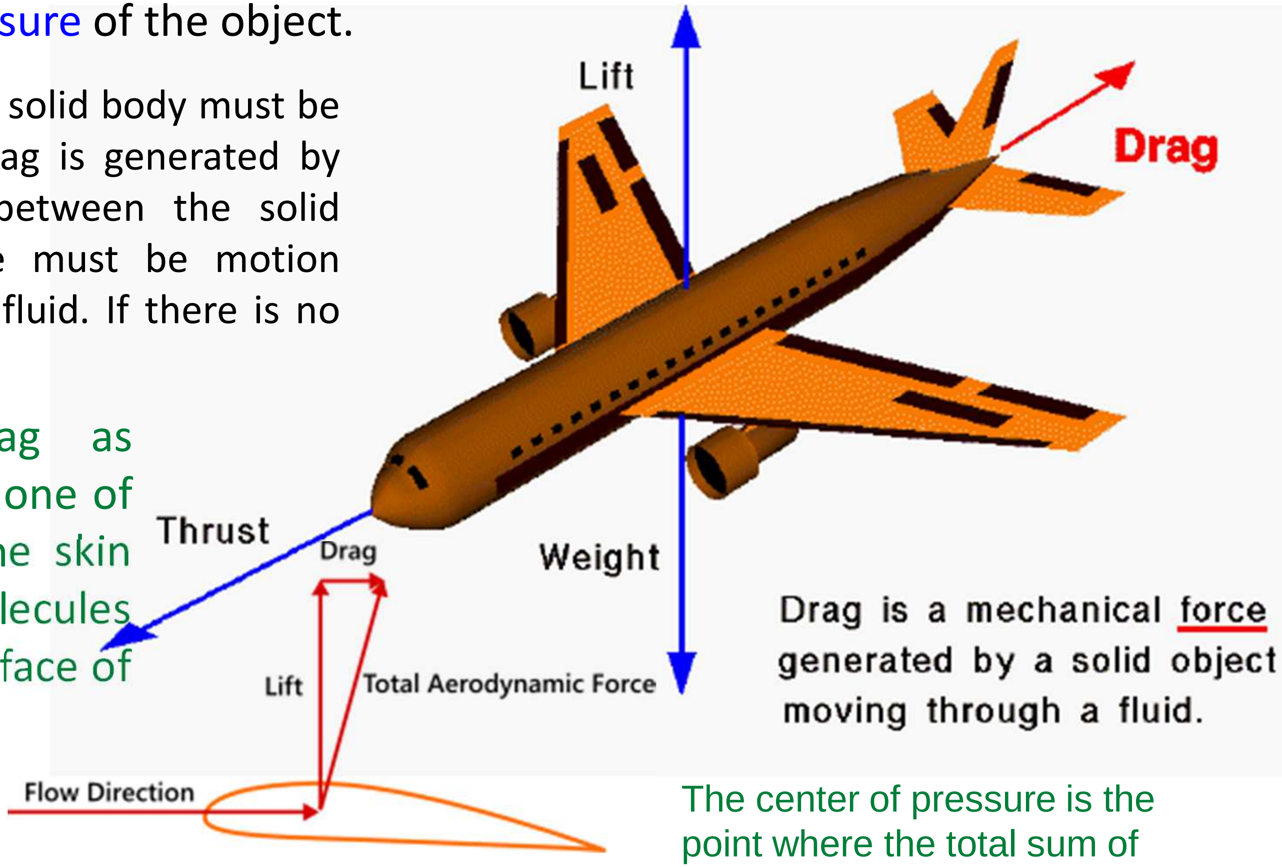
Drag and lift

The component of the aerodynamic force that is opposed to the motion is the drag; the component perpendicular to the motion is the lift. Both the lift and drag force act through the center of pressure of the object.

For drag to be generated, the solid body must be in contact with the fluid. Drag is generated by the difference in velocity between the solid object and the fluid. There must be motion between the object and the fluid. If there is no motion, there is no drag.

We can think of drag as aerodynamic friction, and one of the sources of drag is the skin friction between the molecules of the air and the solid surface of the aircraft.

Mean drag coeff= $C_D = \dfrac{1.328}{\sqrt{\mathrm{Re}}}$



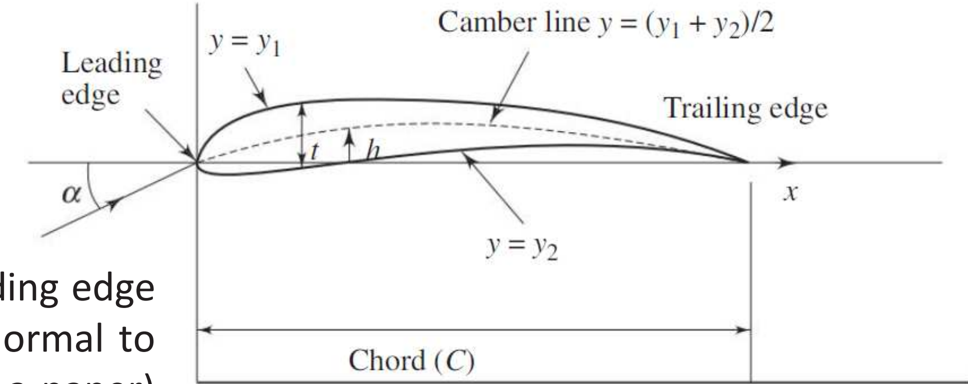
The center of pressure is the point where the total sum of a pressure field acts on a body.

The lift around an immersed body is generated when the flow field possesses circulation (Γ). The lift around a body of any shape is given by $L = \rho U_0 \Gamma$, where ρ is the density and U_0 is the velocity in the streamwise direction.

Flow of Ideal Fluids

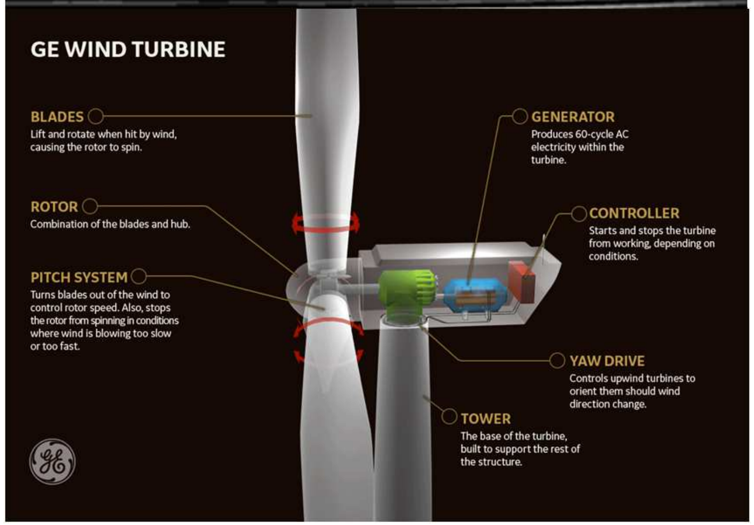
Aerofoil theory

Aerofoils are streamline shaped wings which are used in airplanes and turbomachinery. These shapes are such that the drag force is a very small fraction of the lift.



The chord (c) is the distance between the leading edge and trailing edge. The length of an aerofoil, normal to the cross-section (i.e., normal to the plane of a paper) is called the span of aerofoil.

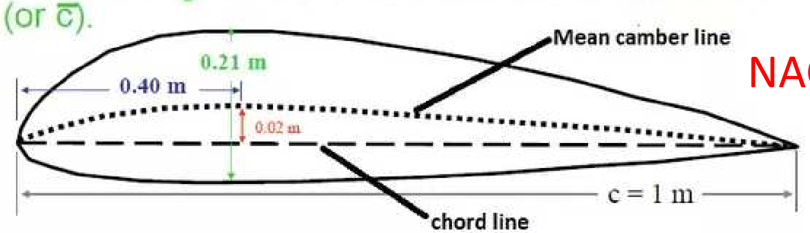
The camber line represents the mean profile of the aerofoil. The ratio of maximum thickness (t) to chord is (t/c) and the ratio of maximum camber (h) (i.e. difference of chord line and mean chamber line) to chord is (h/c).



1st Digit: Maximum camber is 2% of 2D airfoil chord length, c (or 3D wing mean chord length, $\bar{c}$).

2nd Digit: Location of maximum camber is at 4/10ths (or 40%) of the chord line, from the LE.

3rd & 4th Digits: Maximum thickness is 21% of c (or $\bar{c}$).



NACA 2421

Viscous Incompressible Flows

Viscous Incompressible Flows

- In the analysis of motion of a real fluid, the effect of viscosity should be given consideration.
- Influence of viscosity is more pronounced near the boundary of a solid body immersed in a fluid in motion. The relationship between stress and rate of strain for the motion of real fluid flow was first put forward by Sir Isaac Newton and for this reason the viscosity law bears his name.
- Later on, G.G. Stokes, an English mathematician and C.L.M.H. Navier, a French engineer, derived the exact equations that govern the motion of real fluids. These equations are in general valid for compressible or incompressible laminar flows and known as Navier-Stokes equations.
- When a motion becomes turbulent, these equations are generally not able to provide with a complete solution. Usually, in order to obtain accurate results for such situations, the Navier-Stokes equations are modified and solved based on several semi-empirical theories (turbulent models like k-ε, k-ω).

Here we will discuss the equation of motion for laminar flows and various other aspects of laminar incompressible flows.

Viscous Incompressible Flows

NAVIER-STOKES EQUATIONS

Generalized equations of motion of a real flow are named after the inventors of them and they are known as Navier-Stokes equations. However, they are derived from the Newton's second law which states that the product of mass and acceleration is equal to sum of the external forces acting on a body. External forces are of two kinds - one acts throughout the mass of the body and another acts on the boundary.

The first one is known as body force (gravitational force, electromagnetic force) and the second one is surface force (pressure and frictional force).

$$\rho\frac{Du}{Dt}=\rho f_x-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial u}{\partial x}-\frac{2}{3}\nabla.\vec{V}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]\right]$$

$$\rho\frac{Dv}{Dt}=\rho f_y-\frac{\partial p}{\partial y}+\frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial v}{\partial y}-\frac{2}{3}\nabla.\vec{V}\right)+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]\right]$$

$$\rho\frac{Dw}{Dt}=\rho f_z-\frac{\partial p}{\partial z}+\frac{\partial}{\partial z}\left[\mu\left(2\frac{\partial w}{\partial z}-\frac{2}{3}\nabla.\vec{V}\right)+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]\right]$$

Equation of continuity

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

Viscous Incompressible Flows

NAVIER-STOKES EQUATIONS

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In case of incompressible flow

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho f_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho f_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho f_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

temporal convective

Local acceleration

Body force Pressure gradient Viscous term

$$\rho\frac{Du}{Dt}=\rho\vec{f}_b-\nabla p+\mu\nabla^2\vec{V}$$

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\nabla.\vec{V}=0$$

Viscous Incompressible Flows

EXACT SOLUTION OF NAVIER-STOKES EQUATIONS

1. Parallel flow in a straight channel, $u \neq 0$, $v = w = 0$, Flow is independent in the z direction
2. Couette flow : one plate is rest and another plate is moving with a velocity U
3. Hagen Poiseuille flow: fully developed laminar flow through a straight tube of circular cross section
4. Flow between two concentric rotating cylinders
5. Low Reynolds number flow: Theory of hydrodynamics lubrication
6. etc.

Compressible Flow

Compressible Flow

Compressible flow is often called as variable density flow. The compressibility of the fluid, K,

$$K = \frac{1}{\rho}\frac{d\rho}{dp} = -\frac{1}{\nu}\left(\frac{d\nu}{dp}\right)$$

However, when a gas is compressed, its temperature increases. Therefore, the above mentioned definition of compressibility is not complete unless temperature condition is specified. If the temperature is maintained at a constant level, the isothermal compressibility is defined as

$$K_T = \frac{1}{\rho}\frac{d\rho}{dp} = -\frac{1}{\nu}\left(\frac{d\nu}{dp}\right)_T$$

We can say that the proper criterion for a nearly incompressible flow is a small Mach number

$$Ma = \frac{V}{a} << 1$$

where V is the flow velocity and '**a**' is the speed of sound in the fluid. For small Mach number, changes in fluid density are small everywhere in the flow field. In this chapter we shall treat compressible flows which have Mach numbers greater than 0.3 and exhibit appreciable density changes.

- Ma < 0.3: incompressible flow; change in density is negligible.
- 0.3 < Ma < 0.8: subsonic flow; density changes are significant but shock waves do not appear.
- 0.8 < Ma < 1.2: transonic flow; shock waves appear and divide the subsonic and supersonic regions of the flow. Transonic flow is characterized by mixed regions of locally subsonic and supersonic flow.
- 1.2 < Ma < 3.0: supersonic flow; flow field everywhere is above acoustic speed. Shock waves appear and across the shock wave, the streamline changes direction discontinuously.
- 3.0 > Ma: hypersonic flow; where the temperature, pressure and density of the flow increase almost explosively across the shock wave.

Compressible Flow

THERMODYNAMIC RELATIONS OF PERFECT GASES

Compressible flow calculations can be made by assuming the fluid to be a perfect gas. For a perfect gas, it can be written

$$pV\!\!\!\!- = MRT$$

where p is pressure (N/m^2), $V\!\!\!\!-$ is the volume of the system (m^3), M is the mass of the system (kg), R is the specific gas constant (J/kg K) and T is the temperature (K). This equation of state can be written as

$$pv\!\!\!- = RT$$ where $v\!\!\!-$ is the specific volume (m^3/kg).

$$p = \rho RT$$

In another approach, which is particularly useful in chemically reacting systems, the equation of state is written as

$$pV\!\!\!\!- = N\Re T$$ where N is the number of moles in the system, and **R** is the universal gas constant (8314 J/ (kg-mol-K)).

Internal Energy and Enthalpy

Energy of a particle can consist of translational energy, rotational energy, vibrational energy and electronic energy. All these energies summed over all the particles of the gas, form the internal energy '**e**' of the gas.

Let, us imagine a gas is in equilibrium. Equilibrium signifies gradients in velocity, pressure, temperature and chemical concentrations do not exist. Let '**e**' be the internal energy per unit mass. Then the enthalpy '**h**' is defined per unit mass, $h = e + pv\!\!\!-$

Compressible Flow

Internal Energy and Enthalpy

The enthalpy '**h**' is defined per unit mass, $h = e + p\cancel{v}$

$e = e(T, \cancel{v}) \qquad h = h(T, p)$

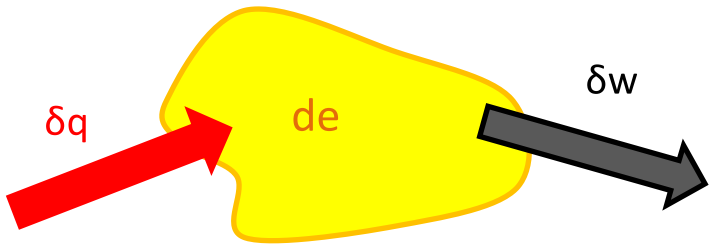
In most of the compressible flow applications, the pressure and temperatures are such that the gas can be considered as calorically perfect (specific heat capacity is a constant value). However, for calorically perfect gases, we can accept constant specific heats and write $c_p - c_v = R$

specific heats at constant pressure and constant volume are defined as

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \qquad c_v = \left(\frac{\partial e}{\partial T}\right)_v$$

Finally

$$c_p = \frac{\gamma R}{\gamma - 1} \qquad c_v = \frac{R}{\gamma - 1} \qquad \gamma = c_p / c_v$$



First Law of Thermodynamics

Let us imagine a system with a fixed mass of gas. If δq amount of heat is added to the system across the system-boundary and if δw is the work done by the system on the surroundings, then there will be an eventual change in internal energy of the system which is denoted by 'de' and we can write $de = \delta q - \delta w$

Here, **de** is an **exact differential** and its value depends only on initial and final states of the system. However, **δq** and **δw** are **dependent on the process**.

Compressible Flow

Entropy and Second Law of Thermodynamics

Entropy: randomness / chaos

First law of thermodynamics does not tell us about the direction (i.e., a hot body with respect to its surrounding will gain temperature or cool down) of the process. To determine the proper direction of a process, we define a new state variable, the entropy, which is

$$ds = \frac{\delta q_{rev}}{T}$$

where s is the entropy of the system, δq_{rev} is the heat added reversibly to the system and T is the temperature of the system.

$$ds = \frac{\delta q}{T} + ds_{irrev}$$

It states that the change in entropy during a process is equal to actual heat added divided by the temperature plus a contribution from the irreversible dissipative phenomena.

The dissipative phenomena always increase the entropy $ds_{irrev} \geq 0$ $\quad ds \geq \frac{\delta q}{T}$

If the process is adiabatic, $\delta q = 0$, $\quad ds \geq 0$

The direction of a process is such that the change in entropy of the system plus surrounding is always positive or zero (for a reversible adiabatic process). In conclusion, it can be said that the second law governs the direction of a natural process.

Isentropic Relation

An isentropic process has already been described as reversible-adiabatic. For an adiabatic process $\delta q = 0$, and for a reversible process, $ds_{irrev} = 0$. We can see that for an isentropic process, $ds = 0$.

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Various statements of the 2nd law:

- Carnot's principle
- Clausius statement
- Kelvin statements
- Kelvin–Planck statement

An adiabatic process is a thermodynamic process which occurs without transferring heat or mass between the system and its surroundings.

Compressible Flow

Fluid density varies significantly due to a large Mach number (Ma = V/a) flow. This leads to a situation where continuity and momentum equations must be coupled to the energy equation and the equation of state to solve for the four unknowns, namely, p, r, T and V.

A nozzle is basically a converging or converging-diverging duct where the kinetic energy keeps increasing at the expense of static pressure. A diffuser has a reversed geometry where pressure recovery takes place at the expense of kinetic energy. At supersonic velocities, the normal-shock wave appears across which the gas discontinuously reverts to the subsonic conditions.

In order to understand the effect of non-isentropic flow conditions, an understanding of constant area duct flow with friction and heat transfer is necessary. These are known as Fanno line flows and Rayleigh line flows, both of which entail choking of the exit flow. The conditions before and after a normal shock are defined by the points of intersection of Fanno and Rayleigh lines on a T-s diagram.

If a supersonic flow is made to change its direction, the oblique shock is evolved. The oblique shock continues to bend in the downstream direction until the Mach number of the velocity component normal to the wave is unity.