Lecture
on
Sub: Fluid Machines
Code : MCC 16101
Topic : Jet Striking Plate

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17.1 INTRODUCTION

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion or from impulse-momentum equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

17.2 FORCE EXERTED BY THE JET ON A STATIONARY VERTICAL PLATE

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in Fig. 17.1

Let V = velocity of the jet, d = diameter of the jet,

$$a = \text{area of cross-section of the jet} = \frac{\pi}{4} d^2.$$

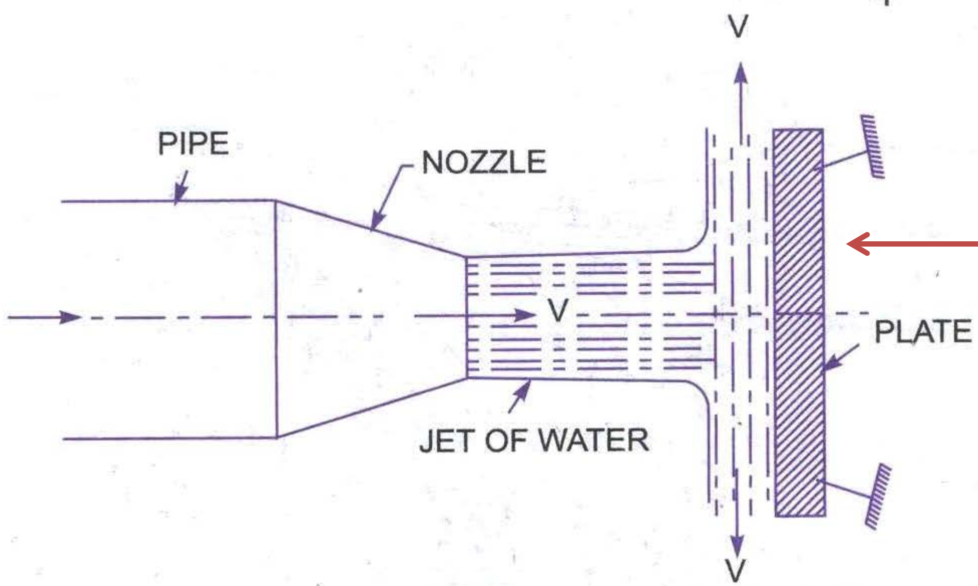


Fig. 17.1 *Force exerted by jet on vertical plate.*

Half of the mass is going in one direction and rest in another direction, by adding these two total mass is obtained.

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

$$F_x = \text{Rate of change of momentum in the direction of force}$$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= (\text{Mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$$

$$= \rho a V[V - 0] \qquad (\because \quad \text{mass/sec} = \rho \times a\,V)$$

$$= \rho a V^2 \qquad \text{...(17.1)}$$

For deriving above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet is to be calculated then final minus initial velocity is taken. But if the force exerted by the jet on the plate is to be calculated, then initial velocity minus final velocity is taken.

Note. In equation (17.1), if the value of density (ρ) is taken in S.I. units (*i.e.*, kg/m^3), the force (F_x) will be in Newton (N). The value of ρ for water in S.I. units is equal to 1000 kg/m^3.

17.2.2 Force Exerted by a Jet on Stationary Curved Plate

(A) Jet strikes the curved plate at the centre. Let a jet of water strikes a fixed curved plate at the centre as shown in Fig. 17.3. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at outlet of the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of jet $= -V\cos\theta$.

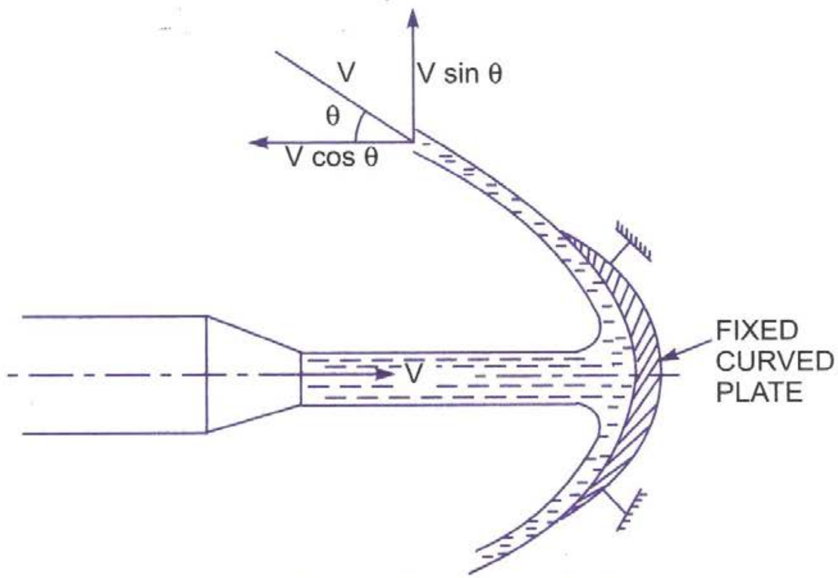


Fig. 17.3 *Jet striking a fixed curved plate at centre.*

(–ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle).

Component of velocity perpendicular to the jet $= V\sin\theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity in the direction of jet $= V$

V_{2x} = Final velocity in the direction of jet $= -V\cos\theta$

$$\therefore \qquad F_x = \rho a V[V - (-V\cos\theta)] = \rho a V[V + V\cos\theta]$$
$$= \rho a V^2[1 + \cos\theta] \qquad \text{...(17.5)}$$

Similarly, $\qquad F_y = \text{Mass per sec} \times [V_{1y} - V_{2y}]$

where $\quad V_{1y}$ = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V\sin\theta$

$$\therefore \qquad F_y = \rho a V[0 - V\sin\theta] = -\rho a V^2 \sin\theta \qquad \text{...(17.6)}$$

–ve sign means that force is acting in the downward direction. In this case the angle of deflection of the jet $= (180° - \theta)$ ...[17.6 (*A*)]

(B) Jet strikes the curved plate at one end tangentially when the plate is symmetrical. Let the jet strikes the curved fixed plate at one end tangentially as shown in Fig. 17.4. Let the curved plate is symmetrical about *x*-axis. Then the angle made by the tangents at the two ends of the plate will be same.

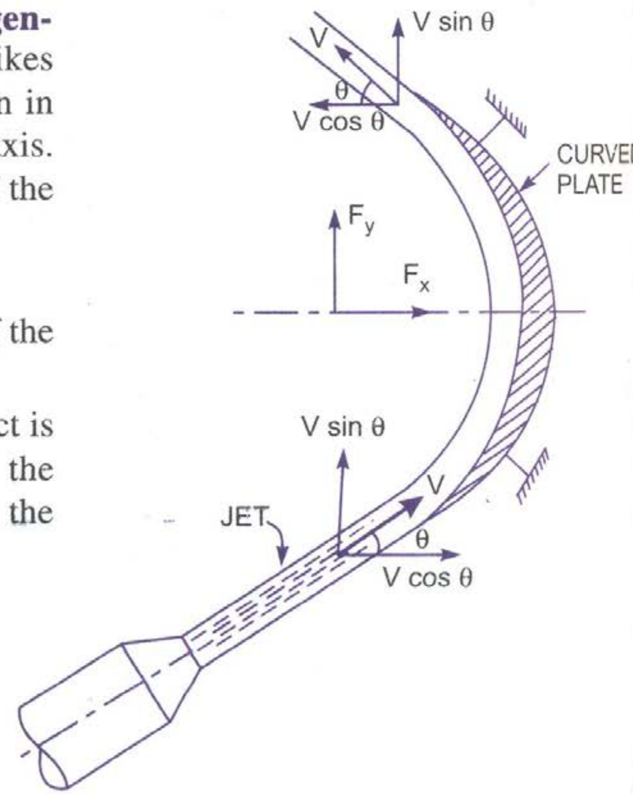


Fig. 17.4 *Jet striking curved fixed plate at one end.*

Let $\quad V$ = Velocity of jet of water,

θ = Angle made by jet with x-axis at inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the directions of x and y are

$$F_x = (\text{mass/sec}) \times [V_{1x} - V_{2x}]$$
$$= \rho a V[V\cos\theta - (-V\cos\theta)]$$
$$= \rho a V[V\cos\theta + V\cos\theta]$$
$$= 2\rho a V^2 \cos\theta \qquad \text{...(17.7)}$$
$$F_y = \rho a V[V_{1y} - V_{2y}]$$
$$= \rho a V[\,V\sin\theta - V\sin\theta\,] = 0$$

17.4.1 Force on Flat Vertical Plate Moving in the Direction of Jet.

Fig. 17.10 shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity of the jet (absolute),

a = Area of cross-section of the jet,

u = Velocity of the flat plate.

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

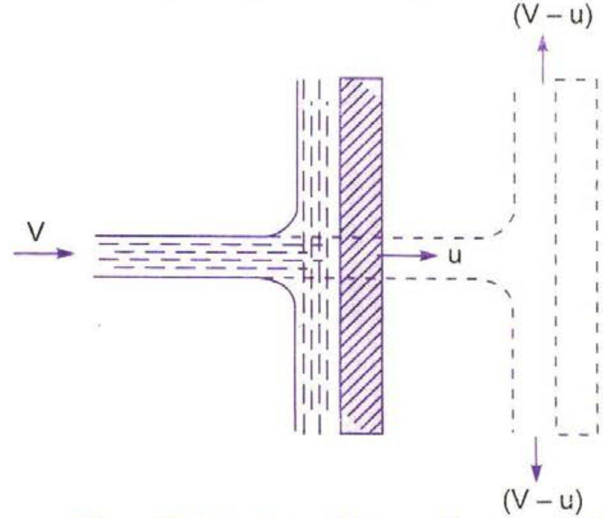


Fig. 17.10 *Jet striking a flat vertical moving plate.*

Hence relative velocity of the jet with respect to plate

$= (V - u)$

Mass of water striking the plate per sec

$= \rho \times$ Area of jet $\times$ Velocity with which jet strikes the plate

$= \rho a \times [V - u]$

$\therefore$ Force exerted by the jet on the moving plate in the direction of the jet,

F_x = Mass of water striking per sec $\times$ [Initial velocity with which water strikes – Final velocity]

$= \rho a(V - u)\,[(V - u) - 0]$ ($\because$ Final velocity in the direction of jet is zero)

$$= \rho a(V - u)^2 \quad \text{...(17.11)}$$

In this case, the work will be done by the jet on the plate, as plate is moving. For the stationary plates, the work done is zero.

$\therefore$ Work done per second by the jet on the plate

$$= \text{Force} \times \frac{\text{Distance in the direction of force.}}{\text{Time}}$$

$$= F_x \times u = \rho a(V - u)^2 \times u \quad \text{...(17.12)}$$

In equation (17.12), if the value of ρ for water is taken in S.I. units (*i.e.*, 1000 kg/m^3), the work done will be in N m/s. The term $\frac{\text{'Nm'}}{\text{s}}$ is equal to W (watt).

17.4.3 Force on the Curved Plate when the Plate is Moving in the Direction of Jet.

Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.12.

Let V = Absolute velocity of jet,

a = Area of jet,

u = Velocity of the plate in the direction of the jet.

Relative velocity of the jet of water or the velocity with which jet strikes the curved plate = $(V - u)$.

If plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane = $(V - u)$.

This velocity can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of the jet.

Component of the velocity in the direction of jet

$$= -(V - u)\cos\theta$$

(–ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of the velocity in the direction perpendicular to the direction of the jet = $(V - u)\sin\theta$.

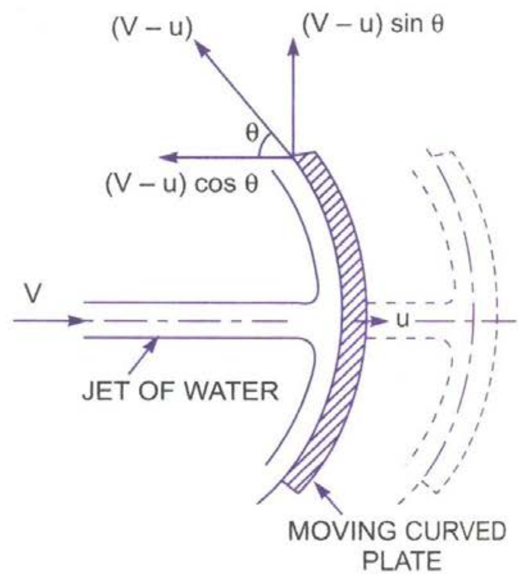


Fig. 17.12 *Jet striking a curved moving plate.*

Mass of the water striking the plate = $\rho \times a \times$ Velocity with which jet strikes the plate

$$= \rho a(V - u)$$

$\therefore$ Force exerted by the jet of water on the curved plate in the direction of the jet,

F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet – Final velocity]

$$= \rho a(V - u)\,[(V - u) - (-(V - u)\cos\theta)]$$

$$= \rho a(V - u)\,[(V - u) + (V - u)\cos\theta]$$

$$= \rho a(V - u)^2\,[1 + \cos\theta] \qquad \text{...(17.17)}$$

Work done by the jet on the plate per second

$$= F_x \times \text{Distance travelled per second in the direction of } x$$

$$= F_x \times u = \rho a(V - u)^2\,[1 + \cos\theta] \times u$$

$$= \rho a(V - u)^2 \times u\,[1 + \cos\theta] \qquad \text{...(17.18)}$$

17.4.5 Force Exerted by a Jet of Water on a Series of Vanes. The force exerted by a jet of water on a *single* moving plate (which may be flat or curved) is not practically feasible. This case is only a theoretical one. In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart as shown in Fig. 17.22. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate. Thus each plate appears successively before the jet and the jet exerts force on each plate. The wheel starts moving at a constant speed.

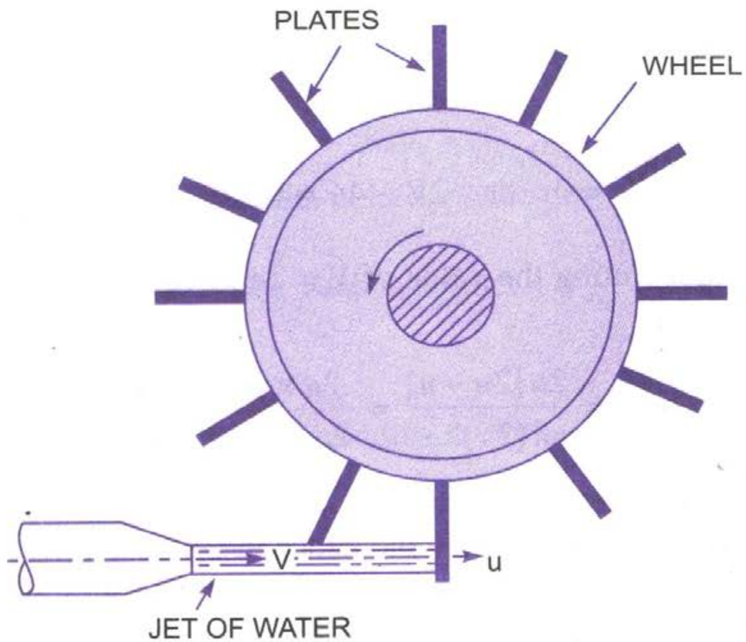


Fig. 17.22 *Jet striking a series of vanes.*

V = Velocity of jet,

d = Diameter of jet,

a = Cross-sectional area of jet,

$= \frac{\pi}{4} d^2$

u = Velocity of vane.

In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered. Hence mass of water per second striking the series of plates = ρaV.

Also the jet strikes the plate with a velocity = $(V - u)$.

After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero.

$\therefore$ The force exerted by the jet in the direction of motion of plate,

$$F_x = \text{Mass per second [Initial velocity – Final velocity]}$$

$$= \rho aV[(V - u) - 0] = \rho aV[V - u] \qquad \text{...(17.22)}$$

Work done by the jet on the series of plates per second

$$= \text{Force} \times \text{Distance per second in the direction of force}$$

$$= F_x \times u = \rho aV[V - u] \times u$$

Kinetic energy of the jet per second

$$= \frac{1}{2} mV^2 = \frac{1}{2} (\rho aV) \times V^2 = \frac{1}{2} \rho aV^3$$

$\therefore$ Efficiency,

$$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho aV[V-u] \times u}{\frac{1}{2}\rho aV^3} = \frac{2u[V-u]}{V^2} \qquad \text{...(17.23)}$$

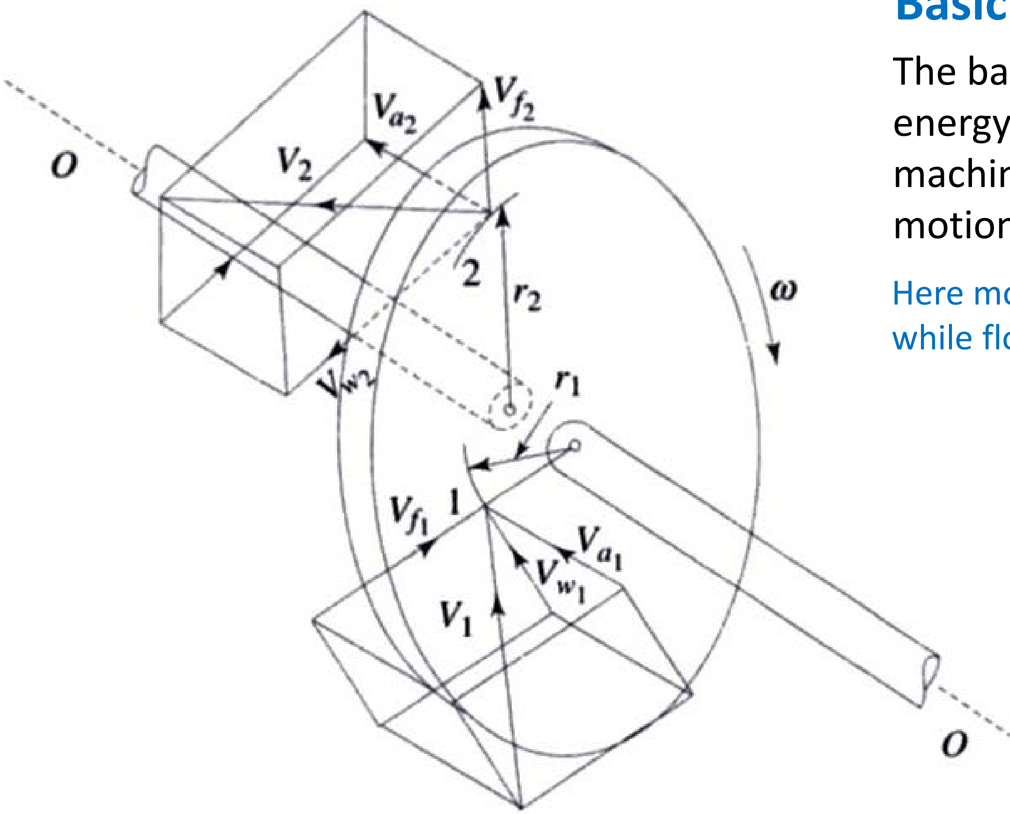
Condition for Maximum Efficiency. Equation (17.23) gives the value of the efficiency of the wheel. For a given jet velocity V, the efficiency will be maximum when

$$\frac{d\eta}{du} = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V-u)}{V^2}\right] = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2uV - 2u^2}{V^2}\right] = 0$$

or

$$\frac{2V - 2 \times 2u}{V^2} = 0 \quad \text{or} \quad 2V - 4u = 0 \quad \text{or} \quad V = \frac{4u}{2} = 2u \quad \text{or} \quad u = \frac{V}{2}. \qquad \text{...(17.24)}$$

Theory of Rotodynamic Machines



Basic Equations:

The basic equation of fluid dynamics relating to energy transfer is same for all rotodynamic machines and is a simple form of Newton's Law of motion applied to fluid element traversing a rotor.

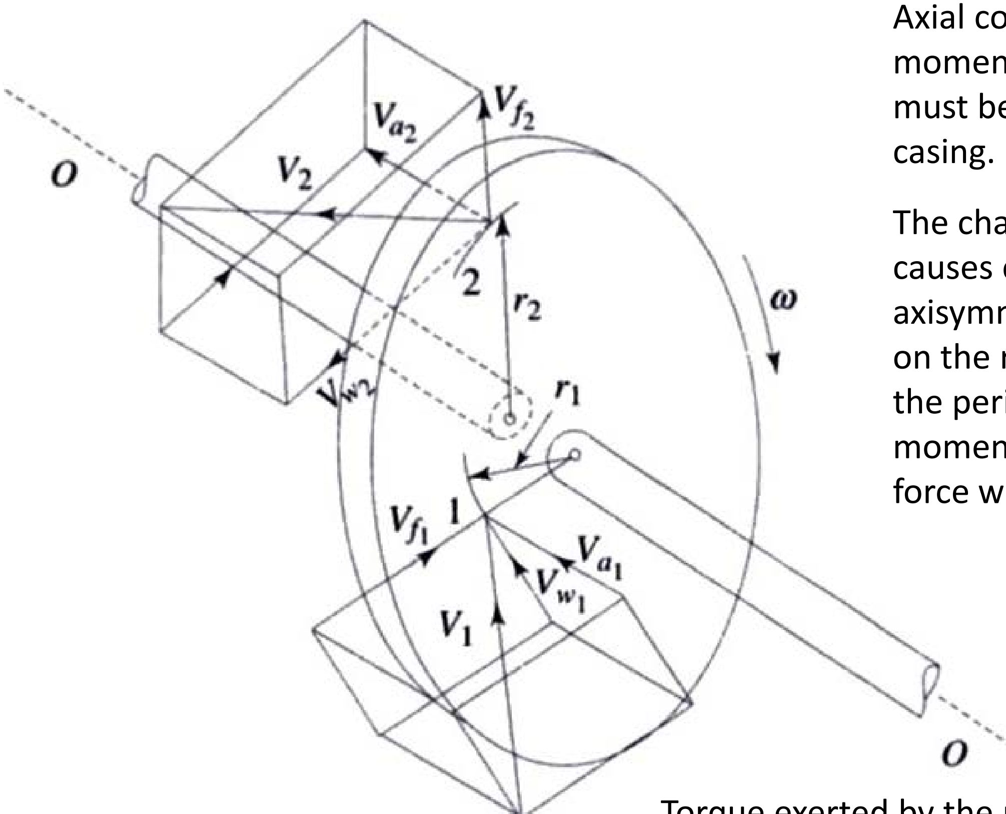
Here momentum theory is applicable to a fluid element while flowing through fixed and moving vanes.

Here o-o is the axis of rotation, ω is angular velocity, fluid enters the rotor with an absolute velocity V at 1 and exit from the rotor from 2. the point 1 and 2 are r_1 and r_2 distance apart from the axis of rotation. The direction of fluid velocities are at arbitrary angle. For analysis of energy transfer due to fluid flow in this situation, following assumptions are considered.

1) The flow is steady, i.e. mass flow rate is constant across any section i.e. no storage or depletion of fluid mass in the rotor.
2) heat and work interactions between the rotor and its surroundings take place at a constant rate.
3) The velocity is uniform over any area normal to the flow, i.e. velocity vector at any point is representative of total flow over a finite area.
4) No leakage loss and entire fluid is undergoing the same process

Here axial component of velocity V_a is parallel to the axis of rotation, radial component of velocity V_f is directed radially through the axis of rotation, tangential component V_w is directed at right angles to the radial direction and along the tangent to the rotor at the part.

Theory of Rotodynamic Machines



Axial component of velocity V_a causes a change in the axial momentum. This change gives rise to an axial force which must be taken by thrust bearing to the stationary rotor casing.

The change in magnitude in radial component of velocity V_f causes change in momentum in radial direction. For an axisymmetric flow, this does not result in any net radial force on the rotor. In case of non-uniform flow distribution over the periphery of the rotor in practice, a change in momentum in the radial direction may result in a net radial force which is carried as a journal load.

The tangential component V_w has an effect on the angular motion of the rotor. In consideration of entire fluid body within the rotor as a control volume, following moment of momentum theorem can be written.

Torque exerted by the rotor on the moving fluid is $T = m\left(V_{w2}r_2 - V_{w1}r_1\right)$

Rate of energy transfer to the fluid is $E = T\omega = m\omega\left(V_{w2}r_2 - V_{w1}r_1\right) = m\left(V_{w2}u_2 - V_{w1}u_1\right)$

Rate of energy transfer to the fluid per unit mass of fluid is $= \left(V_{w2}u_2 - V_{w1}u_1\right)$

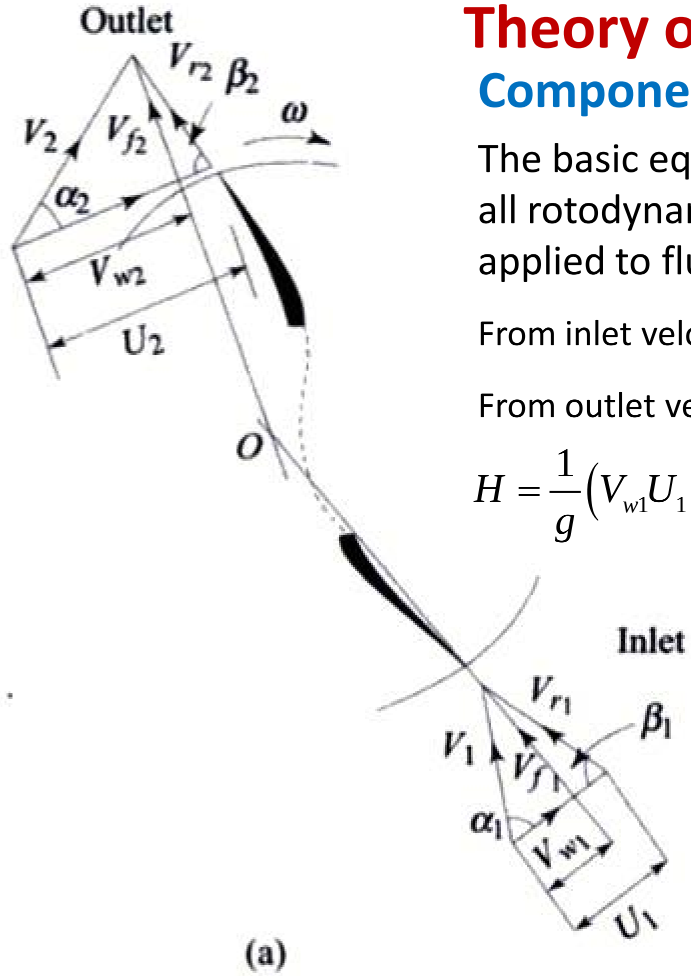
Rate of energy transfer to the fluid per unit weight of fluid or Euler Head is $H = \left(V_{w2}u_2 - V_{w1}u_1\right)/g$

Euler's Equation

In usual convention relating to fluid machines, the head delivered by the fluid to the rotor is considered to be positive and vice versa. Accordingly the Rate of energy transfer from the fluid to the rotor per unit weight of fluid is ……

$$H = \frac{\left(V_{w1}u_1 - V_{w2}u_2\right)}{g}$$

The above equation is applicable to change in density and component of velocity in other direction. Shape of the path of fluid has no consequence, i.e. Independent of path. It is only dependent on the inlet and outlet condition.



(a)

Theory of Rotodynamic Machines

Components of Energy Transfer:

The basic equation of fluid dynamics relating to energy transfer is same for all rotodynamic machines and is a simple form of Newton's Law of motion applied to fluid element traversing a rotor.

From inlet velocity triangle .... $V_{r1}^2 = V_1^2 + U_1^2 - 2V_1 U_1 \cos(\alpha_1)$ | $V_1 \cos(\alpha_1) = V_{w1}$

From outlet velocity triangle .... $V_{r2}^2 = V_2^2 + U_2^2 - 2V_2 U_2 \cos(\alpha_2)$ | $V_2 \cos(\alpha_2) = V_{w2}$

$$H = \frac{1}{g}\left(V_{w1}U_1 - V_{w2}U_2\right) = \frac{1}{2g}\left\{\underbrace{\left(V_1^2 - V_2^2\right)}_{\text{Dynamic Head (Term I)}} + \underbrace{\left(U_1^2 - U_2^2\right)}_{\text{Centrifugal Head (Term II)}} - \underbrace{\left({V_{r2}}^2 - {V_{r1}}^2\right)}_{\text{Pressure head (Term III)}}\right\}$$

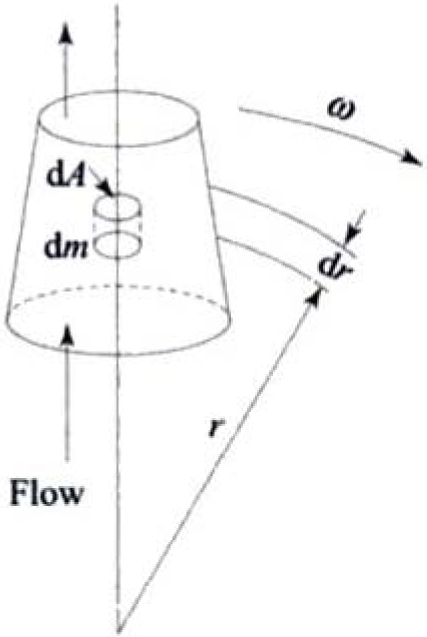
Static Head (Term II + Term III)

The Third term represents a change in the static head due to a change in fluid velocity relative to rotor. This is similar to what happens in case of a flow through a fixed duct of variable cross-sectional area.

Regarding the effect of flow area on fluid through the rotor increases the relative velocity V_r relative to the rotor, converging passage in the direction of flow through the rotor increases the relative velocity $V_{r2} > V_{r1}$ and static pressure decreases based on continuity and Bernoulli's theorem. This case happens with turbine. Opposite cases happens for pump and compressors.

Theory of Rotodynamic Machines

Components of Energy Transfer:



The second term represents a change in fluid energy due to movement of fluid from one radius of rotation to another. This can be explained by demonstrating a steady flow through a container having uniform angular velocity ***ω***. Centrifugal force on an infinitesimal body of a fluid of mass ***dm*** at radius ***r*** gives rise to a pressure difference ***dp*** across the thickness ***dr*** of the body in a manner that a differential force of ***dpdA*** acts on the body radially inward ………. $F = (p + dp)dA - pdA = dpdA$

Centrifugal force …. $Fc = dm\omega^2 r = F$ Or $dpdA = dm\omega^2 r = (\rho dAdr)\omega^2 r$

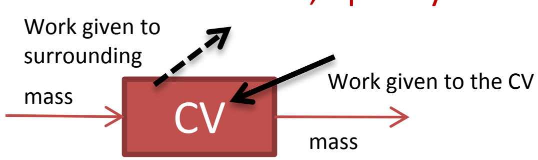
Or $\dfrac{dp}{\rho} = \omega^2 rdr$ For a reversible flow (flow without friction and dissipative effect)

Between two points 1 and 2, the work done per unit mass of fluid (i.e. flow work) can be written as …..

$$\int_1^2 \frac{dp}{\rho} = \int_1^2 \omega^2 rdr = \frac{\omega^2 r_2^{\,2} - \omega^2 r_1^{\,2}}{2} = \frac{U_2^{\,2} - U_1^{\,2}}{2}$$

This work is done on or by the fluid element due to its displacement from radius r1 to r2 and hence becomes equal to energy held or lost by it. Thus centrifugal force is responsible for this energy transfer.

For a control volume, open system …. In light of thermodynamics for adiabatic reversible i.e. isentropic process…



$$u_1 + p_1 v_1 + \frac{V_1^{\,2}}{2} + gz_1 = u_2 + p_2 v_2 + \frac{V_2^{\,2}}{2} + gz_2 + W$$

$$h = u + pv$$

Where ***u*** is internal energy, ***p*** is pressure energy, ***v*** is specific volume, and ***h*** is enthalpy, ***s*** is entropy ***gz*** is potential energy ……. Considering potential and kinetic energy negligible compared to the ***h*** the equation can be rewritten as …

$W = h_1 - h_2$ Enthalpy difference produce work to the system…….   For a isentropic process… $\cancelto{0}{Tds} = dh - vdp$

$$\int_1^2 vdp = \int_1^2 dh \quad \text{Or} \quad \int_1^2 \frac{dp}{\rho} = h_2 - h_1 = -W \quad \text{Or} \quad \frac{U_2^{\,2} - U_1^{\,2}}{2} = h_2 - h_1 = -W$$

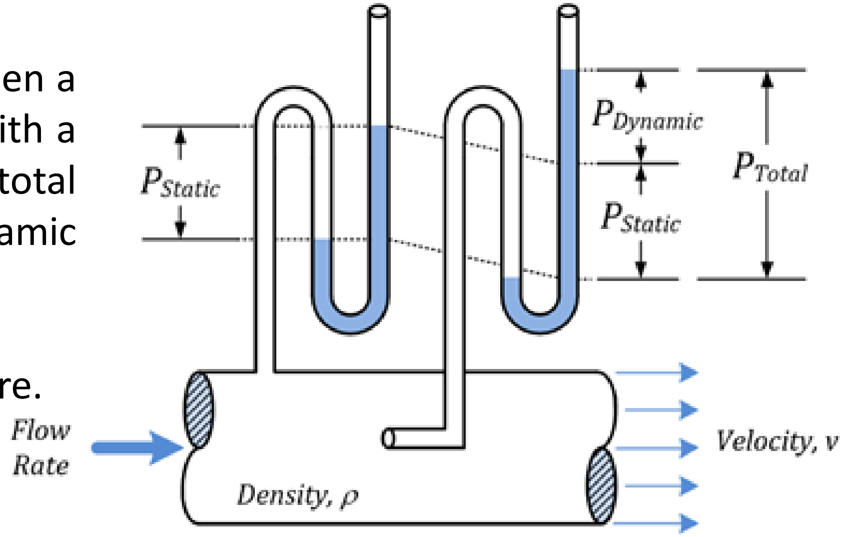
i.e. Flow work

Total Pressure

The total pressure is the force per unit area that is felt when a flowing fluid is brought to rest and is usually measured with a pitot tube type instrument, shown in Figure. The total pressure is the sum of the static pressure and the dynamic pressure.

$P_{total}=P_{static}+P_{dynamic}$

Total pressure is often referred to as the stagnation pressure.



Static Pressure

Static pressure is felt when the fluid is at rest or when the measurement is taken when traveling along with the fluid flow. It is the force exerted on a fluid particle from all directions, and is typically measured with gauges and transmitters attached to the side of a pipe or tank wall. Since static pressure is what most pressure gauges measure, static pressure is usually what is implied when the term "**pressure**" is used in discussions.

Dynamic Pressure

The difference between the total and static pressure is the dynamic pressure, which represents the kinetic energy of the flowing fluid. Dynamic pressure is a function of the fluid velocity and its density and can be calculated from:

$P_{dynamic}=\rho v^2/2g$

**The equation above is in US units, if calculating $P_{dynamic}$ for Metric units (e.g. Pascals) the acceleration of gravity (g) can be omitted.*