Lecture on

Unit / Dimensionless Quantities

by

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Unit Quantities

Unit Quantities refers to turbine parameters which are obtained when a particular turbine operates under 1 m Head / unit head.

D = constant

$\frac{u}{u'} = \frac{V_f}{V_f'} = \frac{\sqrt{2gH}}{\sqrt{2gH'}}$ Therefore $u \propto \sqrt{H}$ Again $u \propto DN$ therefore $u \propto N$ or $\sqrt{H} \propto N$ or $\frac{\sqrt{H}}{\sqrt{H'}} = \frac{N}{N'}$

If H'=1, N'=N_u ; Therefore... $N_u = N/\sqrt{H}$ **(Unit Speed)**

$$\boxed{\frac{\sqrt{H}}{\sqrt{H'}} = \frac{Q}{Q'}}$$

Unit Discharge : $Q = AV_f \propto D^2V_f \propto V_f \propto \sqrt{H}$

If H'=1, Q'=Q_u ; Therefore... $Q_u = Q/\sqrt{H}$ **(unit discharge)**

Unit Power : $P = \gamma QH \propto QH \propto \sqrt{H}H \propto H^{3/2}$

If H'=1, P'=P_u ; Therefore... **(unit power)** $P_u = P/H^{3/2}$

Specific Speed (Ns) …. for Turbine

D ≠ constant

Speed of a geometrically **"similar"** turbine when it is operating under 1 m Head and producing 1 kW power. We know $u \propto DN$ therefore $D \propto u/N$ again $u \propto \sqrt{H}$ so $D \propto \sqrt{H}/N$

$P \propto QH \propto D^2V_fH \propto \left(H/N^2\right)\sqrt{H}H$ Therefore $P = kH^{5/2}/N^2$

If P=1kW, H=1m ; Then N=N_s ; Therefore... $k = N_s^2$ and finally $N_s = N\sqrt{P}/H^{5/4}$

Specific Speed (Ns) …. for Pump

Speed of a geometrically similar pump when it is generating 1 m Head and utilizing 1 m³/s discharge. $Q \propto D^2V_f \propto \left(H/N^2\right)\sqrt{H}$ Therefore … $Q = kH^{3/2}/N^2$

If Q=1m³/s, H=1m ; Then N=N_{sp} ; Therefore... $k = N_{sp}^2$ and finally $N_{sp} = N\sqrt{Q}/H^{3/4}$

* Basically both dynamic and geometric similarity, i.e., similar velocity triangles , with all relevant velocities proportional to each other.

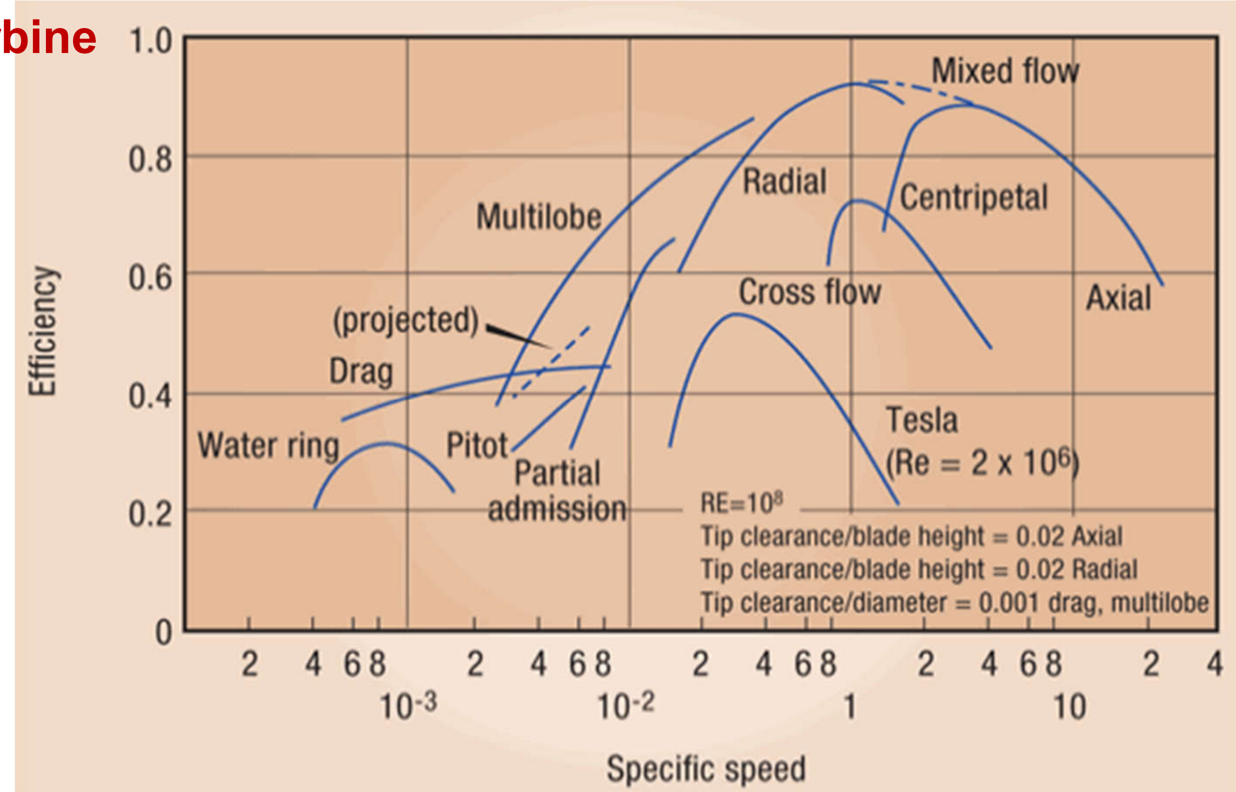
Dimension of Specific Speed for Turbine

energy = force * distance
force = mass * acceleration
acceleration = change of velocity / time
velocity = distance / time
So, Power is M.L^2/T^3
In SI units, Watt = W = kg.m^2/s^3
Therefore, specific speed Ns is

$$Ns = \frac{N\sqrt{P}}{H^{5/4}} = \frac{1}{T}\left(\frac{LM}{T^2}\frac{L}{T}\right)^{\frac{1}{2}} \Big/ L^{\frac{5}{4}}$$

and, dimensionless specific speed Ns' is

$$Ns' = \frac{N\sqrt{P}}{(gH)^{5/4}}$$



Specific Speed Range for Turbine

If P is considered as Hp, Specific Speed (Ns) is in MKS unit.

If P is considered as kW, Specific Speed (Ns) is in SI unit.

Ns (MKS)	Ns(MKS)	Ns(SI)
Pelton with single jet	10-35	8.5-30
Pelton with two or more jet	35-60	30-51
Francis	60-300	51-225
Kaplan	300-1000	225-860

Specific Speed Range for Pump

Centrifugal Pump	Speed type	Specific Speed
Radial flow	Slow Medium High	10 - 30 30 - 50 50 - 80
Mixed flow		80 - 160
Axial flow		100 - 450

Runaway Speed (Nr) for Turbine in terms of Working Speed (N)

It is the maximum speed attained by the runner at a maximum head, at full gate opening when external load (generator) is disconnected from the system and governor is ceases to function.

Turbine Type	Runaway Speed (Nr)
Radial flow	1.8 - 1.9 N
Mixed flow	2 .0 - 2.2 N
Axial flow	2.5 - 3.0 N

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Dimensionless Coefficients

Model → a reduced / enlarged scale structure or machine component

prototype → a full scale structure or machine component

Head Coefficient (C_H)

D ≠ constant

$u = \sqrt{2gH}$ and $u \propto DN$ therefore $\sqrt{H} \propto DN$ or $H \propto D^2 N^2$ therefore

$$\boxed{\frac{H}{D^2 N^2} = \text{constant}}$$

Flow Coefficient (C_F)

$$Q \propto D^2 V_f \quad \propto D^2 \sqrt{H} \quad \propto D^2 (DN) \quad \propto D^3 N$$

$$\boxed{\frac{Q}{D^3 N} = \text{constant}}$$

Power Coefficient (C_P)

$$P \propto D^2 V_f H \quad \propto D^2 \sqrt{H} H \quad \propto D^2 H^{3/2} \quad \propto D^2 (DN)^3 \quad \propto D^5 N^3$$

$$\boxed{\frac{P}{D^5 N^3} = \text{constant}}$$

Operating or Constant Speed Characteristics (in Different gate opening)

For each gate opening speed is kept constant ; discharge Q and head H may vary according to their availability ; the brake power P is measured mechanically by a dynamometer. Overall efficiency η_0 is then calculatcd from the measured values of discharge, head and power. Results are graphically represented by plotting η_0 against %age of full load. The term %age of full load prescribes the ratio of measured power to full load.

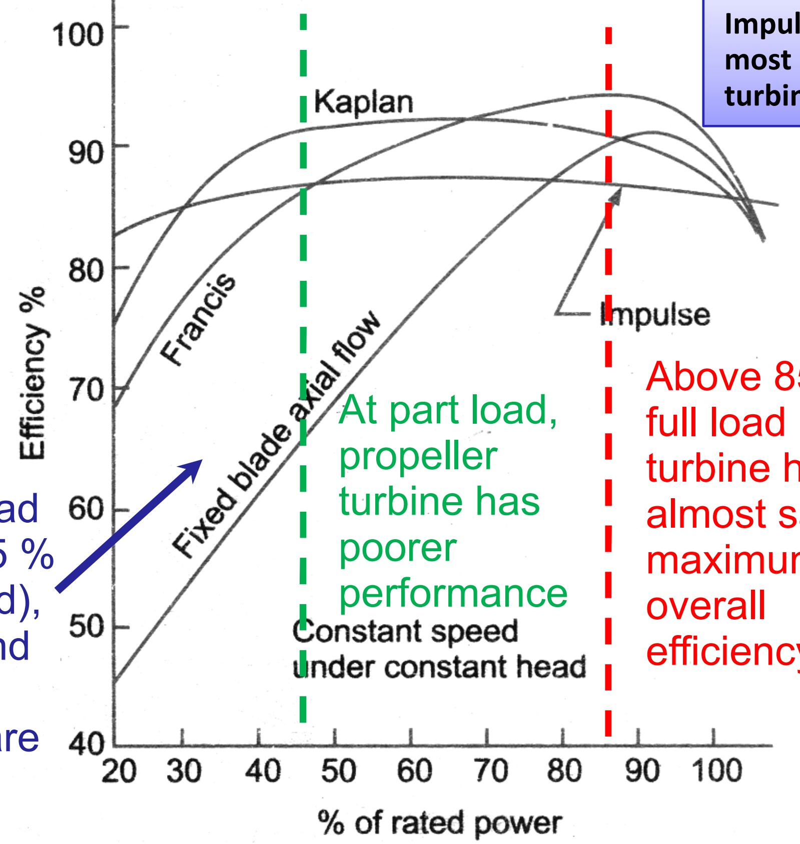
Guide vane is wicket gate

$$\left(\frac{H}{D^2 N^2}\right)_{\text{model}} = \left(\frac{H}{D^2 N^2}\right)_{\text{prototype}}$$

$$\left(\frac{H_m}{D_m{}^2 N_m{}^2}\right) = \left(\frac{H_p}{D_p{}^2 N_p{}^2}\right)$$

Rated power: The power output of a device under a specific or nominal operating conditions

At part load (below 45 % of full load), kaplan and impulse (pelton) are better



At part load, propeller turbine has poorer performance

Above 85% of full load all the turbine have almost same maximum overall efficiency

Impulse is the most consistent turbine

Scale effect

Geometric similarity of a model and prototype cannot be extended to surface roughness. The variation of surface roughness w.r.t size of the turbine will cause a small but appreciable variation in the proportion of the effective head lost due to hydraulic friction.

Thus, the efficiency of the prototype will be different from the corresponding model efficiency. This aspect is called scale effect.

It is observed that with increase in size of a similar mixed or axial flow turbine has greater efficiency that that of the model operating under hydraulically similar condition.

$$\frac{1-\eta_p}{1-\eta_m} = \left(\frac{D_m}{D_p}\right)^{0.2}$$

m- Model → a reduced / enlarged scale structure or machine component

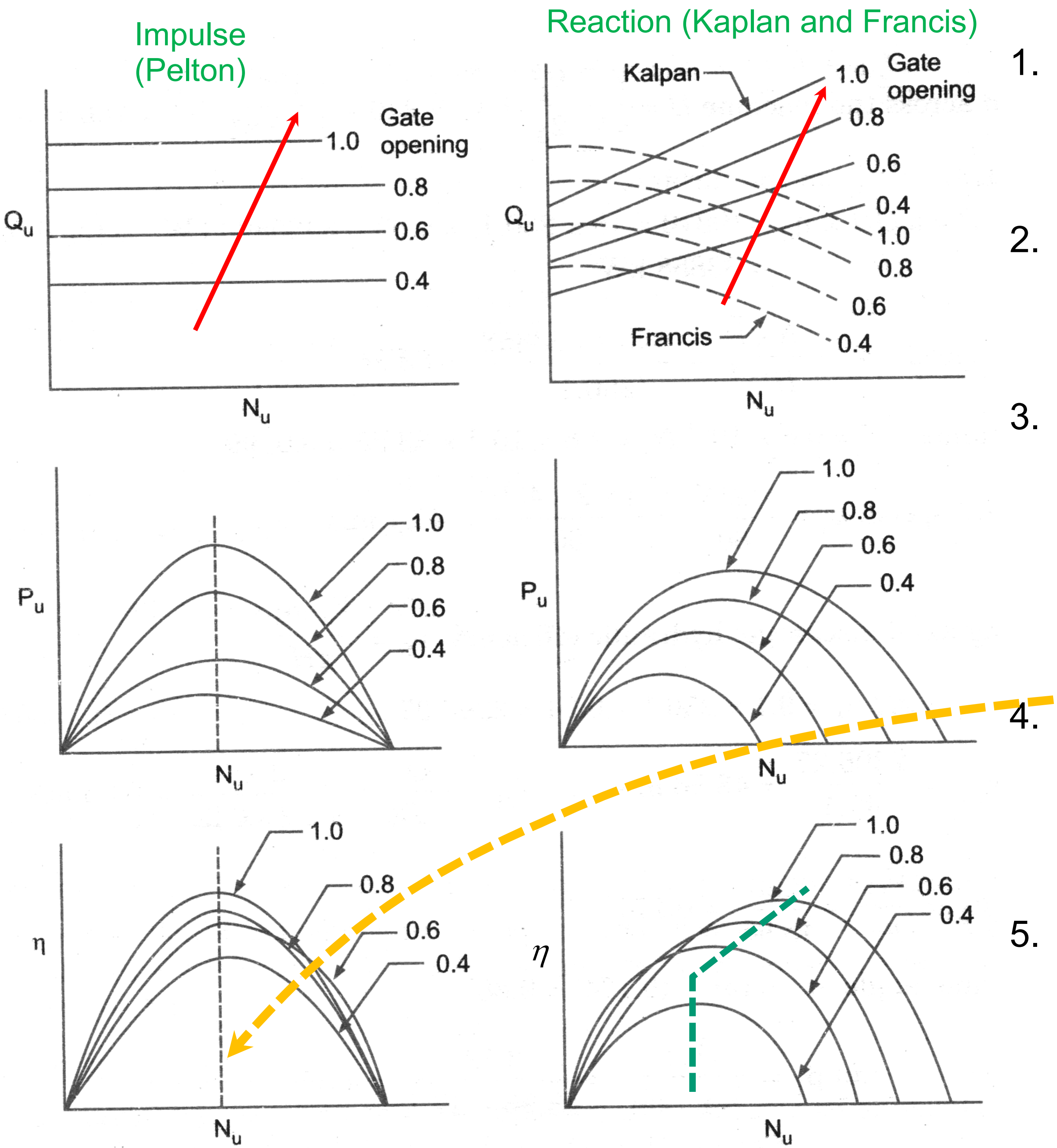
p- prototype → a full scale structure or machine component

Ackert suggested the following formula, considering the friction loss as a function of Reynolds number

$$\frac{1-\eta_p}{1-\eta_m} = \frac{1}{2}\left[1+\left(\frac{D_m}{D_p}\right)^{0.2}\left(\frac{H_m}{H_p}\right)^{0.2}\right]$$

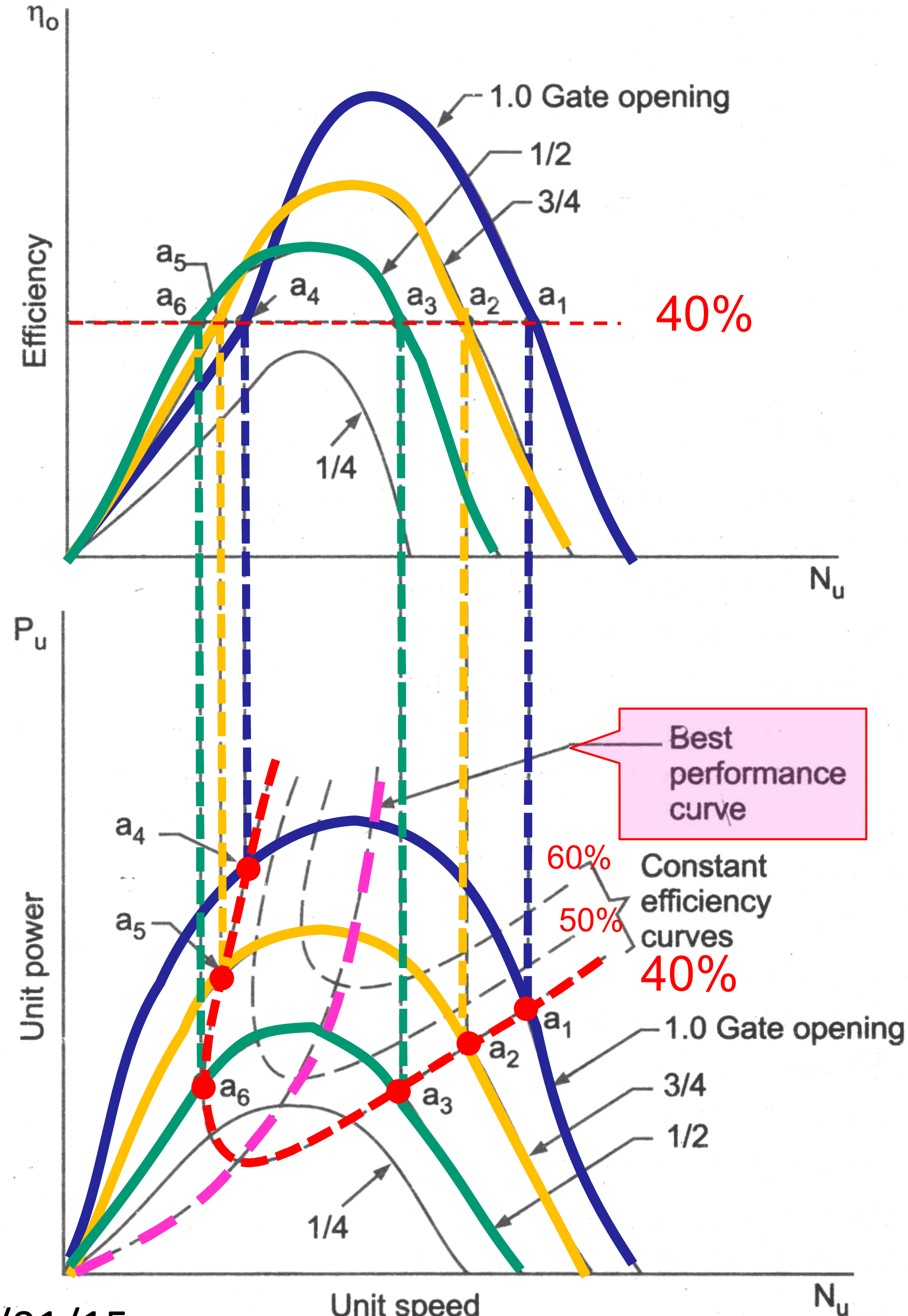
Where, η is overall efficiency, D is linear dimension, H is head

Performance Characteristics



1. Unit discharge curve for a pelton turbine are horizontal indicates that it varies with gate opening but independent in nature.
2. Kaplan→ discharge rises as speed increases, whereas reverse phenomenon occurs in Francis turbine
3. Power and efficiency curve are parabolic in nature, and reach maximum value at a particular speed for pelton turbine. But for reaction turbine it reach maximum value at different speed
4. For pelton, maximum efficiency for different gate opening occurs at same speed, which corresponds to speed ratio ku=0.45 to 0.46
5. In reaction turbine maximum efficiency occurs at different speed.

Muschel Curve (Iso efficiency curve)



1. For a particular efficiency, a horizontal line is drawn which intersect the curve for different gate opening.
2. This information is then transferred to the Pu vs Nu curve for corresponding gate opening.
3. Points of the same efficiency is then joined to get a constant efficiency curve.
4. A curve for best performance is obtained when peak points of various iso-efficiency curves are joined.
5. This curves helps to locate the region where the turbine would operate with maximum efficiency.